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ROOM RESPONSE TO FREQUENCY CHANGE AND ITS RELATION TO THE PITCH CHANGES

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When a signal of constant frequency is sent into a room and its frequency changes to a new value, then, beginning from this moment, in the room there will exist two signals with changing amplitudes (i.e. decreasing one with the old frequency value and an increasing one with a new frequency value). As result of these changes envelope and instantaneous frequency changes appear. These changes have a transient character and exist in a time interval that equals the room reverberation time. Instantaneous frequency and envelope changes are similar to those observed for beating. To describe these changes the modified IWAIF model [1, 2] is used that allows an attempt to evaluate the pitch change in time. It is found that the calculated pitch changes, predicted as an effect of the frequency change in the room, have a monothonical character and appear within the range from the initial frequency to the final one. The rate and character of the calculated predicted pitch changes depends on the amplitude ratio of the two signals, the value of the frequency difference and on the room reverberation time.

1. Introduction

During the superposition of two tones with similar frequencies beating occurs. This phenomenon, elementary from the point of view of physical description, is much more complex from the perceptual point of view. The reason for it are instantaneous frequency (IF) changes which occur simultaneously with the amplitude envelope changes [1, 2, 3, 7]. Moreover, extremal instantaneous frequency changes appear when an amplitude envelope reaches the minimal value [13]. Because of that the perception of instantaneous frequency changes near the envelope minimum is difficult. This problem has already been analysed by Helmholtz. JEFFRESS [7] isolated fragments of beating near the envelope minimum and noticed a considerable difference in pitch for those fragments in comparison with that near the envelope maximum. FETH [3] has studied the pitch for two complementary pair of tones of different frequencies and intensities. The complementarity consisted the fact that the intensity ratios were the same, but for one pair the sound of lower frequency has a higher amplitude than that of higher frequency while for the other one the amplitude relation was reversed. Owing to this fact, the amplitude envelope for both sounds was the same. Pitch differences for both pairs of signals were perceived by subjects. Feth explains the possibility of pitch discrimination for such complementary pairs of signals by the differences in the calculated envelope weighted averaged of instantaneous frequency (EWAIF). DAI [2], basing on the EWAIF model, defined a new pitch measure that is the squared envelope weighted average of instantaneous frequency (SEWAIF), and pointed out its advantage in comparison with the EWAIF model. On the base of his own and other known results of perceptual studies, he showed this measure to correlate better with the SEWAIF model than with the EWAIF one.

ANANTHARAMAN *et al.* [1] applied the IWAIF (intensity weighted average of instantaneous frequency) method for frequency discrimination. The only difference between the SEWAIF and IWAIF models are their names.

The reported studies are closely connected with phenomena observed in rooms for signals of varying frequency. OZIMEK and RUTKOWSKI [10] found some differences between instantaneous frequency changes of a signal received from a room and the original frequency changes of the transmitted signal. According to numerical calculations and experimental investigations in rooms, they proved [14] that for linear and jump frequency changes some additional instantaneous frequency changes appear. The extreme values of instantaneous frequency changes correspond to the minimum values of the amplitude envelope.

The analysis of the instantaneous frequency and envelope changes caused by room transmission properties showed that those changes result from the superposition of waves reaching a selected point with different amplitudes and time delay values. The time delays between particular waves for a sound with varying frequency generate some instantaneous frequency differences. The instantaneous frequency and amplitude differences are the reason of phenomena similar to beating. Now, the main question from the view-point of the sound transmission quality evaluation is whether instantaneous frequency changes in rooms are important for the perception.

In this paper the case of simultaneous instantaneous frequency and envelope changes, appearing due to a sudden step frequency change of a sound in the room are discussed. Those changes are interesting because in a finite time interval after a frequency change in the room there appear two signals of constant frequencies and varying amplitudes. Therefore one should expect results similar to those of beating with effects concerning the perception. The purpose of this work is to predict the possibility of the pitch change resulting a step frequency change of the sound in a room on the basis of the IWAIF [1, 2] model. Frequency changes of this kind appear in real sounds, but they concern rather spectral changes than those of the frequency changes of individual tones.

2. The room response to a frequency change

When a sinusoidal signal of constant amplitude A and frequency ω_1 is sent to a room, the steady state room response is

$$x(t)=A|H(j\omega_1)|\exp[j\varphi(\omega_1)]\sin(\omega_1 t). \tag{2.1}$$

Similarly for a signal of frequency ω_2

$$y(t)=A|H(j\omega_2)|\exp[j\varphi(\omega_2)]\sin(\omega_2 t). \tag{2.2}$$

where $|H(j\omega)|$ is the magnitude of a room transmittance of frequency ω and $\varphi(\omega)$ is the phase shift for this frequency. Let us assume that at a moment $t=0$ the frequency value will change from ω_1 to ω_2. The amplitude of signal transmitted from the source remains constant. Beginning from the moment of the frequency change the amplitude of the signal of frequency ω_1 will decrease according to the function

$$x(t)=A|H(j\omega_1)|\exp[j\varphi(\omega_1)]\exp(-kt)\sin(\omega_1 t). \tag{2.3}$$

At the moment of the frequency change a signal of frequency ω_2 with rising amplitude will appear:

$$y(t)=A|H(j\omega_2)|\exp[j\varphi(\omega_2)][1-\exp(-kt)]\sin(\omega_2 t). \tag{2.4}$$

where $k=13.8/T_{60}$, T_{60} is the room reverberation time for a 60 dB decay. We assume that reverberation time changes slowly with the frequency.

From equations (2.3) and (2.4) it follows that in a practically finite time interval there will be two signals in a room: one decreasing with the frequency ω_1 and another one increasing with the frequency ω^2. In a selected point a resultant signal being the sum of signals (2.3) and (2.4) appear:

$$r(t)=X_0\exp(-kt)\sin(\omega_1 t)+Y_0[1-\exp(-kt)]\sin(\omega_2 t), \tag{2.5}$$

where $X_0=A|H(j\omega_1)|$, $Y_0=A|H(j\omega_2)|$.
To find the resultant signal, the analytic signal corresponding to the sum of signals (2.3) and (2.4) must be created see Appendix. The envelope of the calculated analytic signal (A4 — Appendix) equals that of a real signal.

$$e(t)=X_0\sqrt{D^2(t)+\delta^2 G^2(t)+2\delta D(t)G(t)\cos(\Delta\omega t)}\,. \tag{2.6}$$

The following notations were assumed for simplicity

$$D(t)=\exp(-kt),\ G(t)=1-\exp(-kt),\ \Delta\omega=\omega_2-\omega_1 \quad \text{and} \quad \delta=\frac{Y_0}{X_0}.$$

The knowledge of the resultant signal envelope and of the complex instantaneous phase of the signal CI$\Phi(t)$ (A9 — Appendix) allows to calculate the complex instantaneous frequency (A10 — Appendix) after the frequency change in the room

$$\mathrm{CIF}(t)=\frac{1}{2\pi}\frac{dCI\Phi(t)}{dt}=$$

$$=\frac{1}{2\pi}\left\{\frac{-D(t)\{-\delta^2k+\delta G(t)(k\cos(\Delta\omega t)+\Delta\omega\sin(\Delta\omega t))+kD(t)(1+\delta^2-\delta\cos(\Delta\omega t))\}}{D^2(t)+\delta^2G^2(t)+2\delta D(t)G(t)\cos(\Delta\omega t)}\right\}+$$

$$+j\frac{1}{2\pi}\left\{\omega_1+\left\{\frac{\delta^2\Delta\omega G^2(t)+\delta D(t)[k\sin(\Delta\omega t)+\Delta\omega G(t)\cos(\Delta\omega t)]}{D^2(t)+\delta^2G^2(t)+2\delta D(t)G(t)\cos(\Delta\omega t)}\right\}\right\}. \tag{2.7}$$

The first component of the complex instantaneous frequency (the real part) results from a relative amplitude envelope change. The second component (the imaginary part) equals the sum of the constant value corresponding to the initial frequency and the varying value of the instantaneous frequency IF(t). Using the formula (A11 — Appendix), changes of the complex instantaneous frequency magnitude described by the formula (2.7) as well as the envelope of the resultant signal (2.6) were calculated. The calculations were carried out for an ideal room with an exponential sound decay. Exemplary results of the calculations are shown in the Fig. 1.

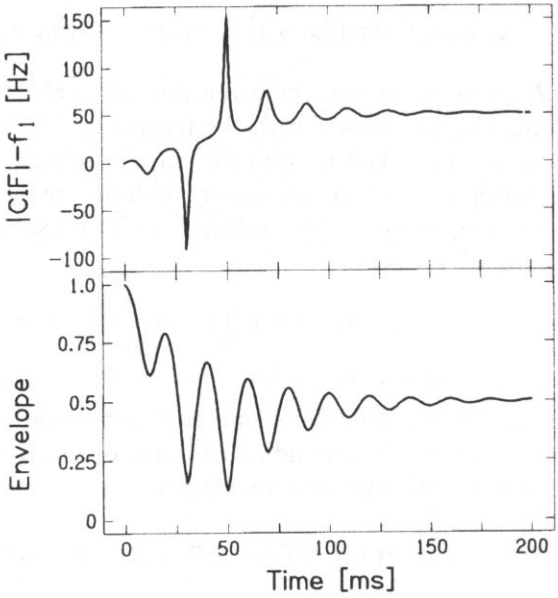

Fig. 1. Exemplary results of the calculated complex instantaneous frequency magnitude and envelope changes after a frequency jump (Δf=50 Hz, δ=0.5, T_{60}=0.5 s).

Figure 1 shows variations in the complex instantaneous frequency magnitude (the top panel) and the relative changes in resultant signal amplitude envelope corresponding to them (bottom panel). The moment of the frequency change corresponds to the zero value on the time axis. At the following moments, specific fluctuations of the complex instantaneous frequency magnitude appear and its steady value corresponds to the frequency ω_2.

For simplicity it was assumed that the initial value of frequency equal zero (which in reality corresponds to the initial frequency ω_1).

An analysis of numerical calculations indicates the following important facts:

The transition from the initial to the final frequency have an oscillating character. These oscillations disappear after the room reverberation time. The frequency of the complex instantaneous frequency magnitude oscillations and of the envelope changes equals (similarly in the case of beating) the value of the frequency difference $\Delta\omega=\omega_2-\omega_1$. The change in the direction of the instantaneous frequency oscillation occurs the moment t_e that is the moment of equilisation of both the signal amplitudes, i.e. it is increasing with the final frequency and decaying with the initial one:

$$t_e=\frac{T_{60}}{13.8}\ln\left(1+\frac{1}{\delta}\right).$$

Around the moment t_e a largest depth of the changes of the amplitude envelope is observed. The value of the instantaneous frequency at t_e equals $f_1+\Delta f/2$.

It is an interesting feature of the complex instantaneous frequency magnitude changes that their extreme values appear at the moments at those the amplitude envelope reaches its minimal values. The observed correlation between instantaneous frequency and envelope changes is similar to that for beating.

Instantaneous frequency changes, very similar to those presented in Fig. 1, can be measured in real rooms. The results of such measurements are presented in [14]. The measured instantaneous frequency changes are more complex because of the approximately exponential sound decay in a real room.

3. Evaluation of the pitch changes as an effect of the frequency change in a room

3.1. The IWAIF model

The expression for the intensity-Weighted Average of Instantaneous Frequency (IWAIF) was defined as a physical measure of the average pitch of complex signals [1, 2]

$$\mathrm{IWAIF}=\frac{\int_0^T e^2(t)f(t)dt}{\int_0^T e^2(t)dt}, \tag{3.1}$$

where $e(t)$ is the amplitude envelope, $f(t)$ is the complex instantaneous frequency magnitude and T is the time interval. When the analytic form of the envelope and instantaneous frequency changes is known from algebraic and trigonometric transformations, the IWAIF calculation is simply. To obtain the envelope and instantaneous frequency changes in the experiments, an appropriate hardware or software, which allows to demodulate the amplitude and frequency is required. The amplitude envelope demodulation is relatively easy to perform, whereas the frequency demodulation of an acoustic signal produces many problems. One of the possible solutions of frequency demodulation was mentioned in an earlier paper [9]. Avoiding technical details, the method of demodulation presented there consists in the instantaneous frequency evaluation by measuring time intervals corresponding to the following zero crossings of a signal in the same direction. The varying in time zero-crossing frequency ZCF is defined

$$\mathrm{ZCF}(t)=\frac{1}{\Delta\tau(t)}, \tag{3.2}$$

where $\Delta\tau$ is a time interval between the following zero-crossings of a tested signal. The measurements of the instantaneous frequency defined in this way can be performed with high precision.

For a signal having a finite number of spectral components the equation (3.1) takes the form

$$\mathrm{IWAIF}=\frac{\sum_{i=1}^{N} a_i^2 f_i}{\sum_{i=1}^{N} a_i^2}, \tag{3.3}$$

where a_1^2 is the amplitude of the i-th intensity spectrum component, f_i is the frequency, and N is the number of components. The value of IWAIF means the frequency ordinate for the center of gravity of a figure created by the signal intensity spectrum. It is very easy to calculate the value of IWAIF for beating because it concerns only two spectral components.

As reported in papers [2, 4, 7] IWAIF for beating has values lying within the frequency range between the spectral components of the beating tones. If the intensities of both components are the same, IWAIF equals the arithmetic average of their frequencies. When the intensities of both components are different, the value of IWAIF shifts in the direction of the component with higher intensity. Feth *et al.* [4] proved that IWAIF well corresponds to the perceived average pitch if the frequency distance between the spectral components does not exceed a critical band. When the difference in the frequencies of these tones is larger than the critical band [12], we perceive not the average pitch but two separate tones of constant intensities and different pitches characteristic of them. However, when both beating tones have similar frequencies, only one tone of intermediate pitch and varying intensity is perceived. Such a perception of the pitch is determined by resonance properties of a basiliar membrane. When the stimulation of a basiliar membrane is performed by two tones lying relatively far from each other on the frequency scale, the separate (non interlaced) areas on the basiliar membrane corresponding to them are stimulated. For a small frequency difference the stimulated resonance areas on the basiliar membrane overlap each other, so that an intermediate pitch is perceived without the possibility to discriminate the pitches of the particular tones.

3.2. Intensity weighted instantaneous frequency magnitude after a frequency change

In the case of the frequency change in a room, starting from the moment of its appearance there will be two spectral components of constant frequency values f_1 and f_2 but with changing in time amplitudes $a(f_1,t)$ and $a(f_2,t)$. The amplitude variability of both the signals differentiate the considered case from beating. As a result of an amplitude variability the evaluated IWAIF value depends on time and can be described by

$$\mathrm{IWIF}(t)=\frac{a_1^2(t)f_1+a_2^2(t)f_2}{a_1^2(t)+a_2^2(t)}. \tag{3.4}$$

Replacing IWAIF by the IWIF results from the signals parameter variability. Now we are not interested in the averaged but in the momentary changes of the amplitude weighted magnitude of the complex instantaneous frequency. After transformation formula (3.4) can be rewritten in the form

$$\mathrm{IWIF}(t)=f_1+\frac{\delta^2(t)}{1+\delta^2(t)}\Delta f, \tag{3.5}$$

where $\delta(t)$ is the ratio of the varying in time amplitudes, $a_2(t)/a_1(t)$, and Δf is the frequency difference f_2-f_1 (the value of a frequency change). Taking into account the real form of the signal amplitude changes for the frequencies f_1 and f_2 after a frequency change in the room ((2.3),(2.4)) we get finally

$$\mathrm{IWIF}(t)=f_1+\frac{\left(\frac{1-\exp(-kt)}{\exp(-kt)}\right)^2}{1+\left(\frac{1-\exp(-kt)}{\exp(-kt)}\right)^2}\Delta f. \tag{3.6}$$

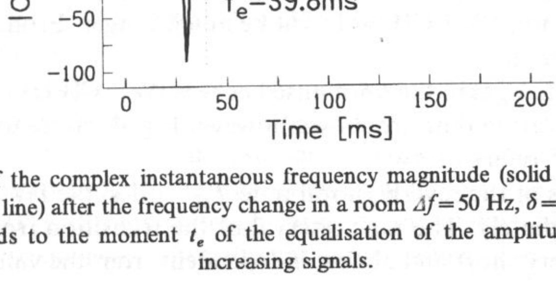

Fig. 2. Comparison of the complex instantaneous frequency magnitude (solid line) and pitch changes IW | CIF(t) | (dashed line) after the frequency change in a room $\Delta f=50$ Hz, $\delta=0.5$, $T_{60}=0.5$ s). Vertical dashed line corresponds to the moment t_e of the equalisation of the amplitude of the decaying and increasing signals.

In Fig. 2 the changes in the complex instantaneous frequency magnitude | CIF | $-f_1$ (solid line) are compared with changes in the function IW | CIF(t) | $-f_1$ (dashed line). The moment of the amplitude equalisation of both signals is marked by a vertical dotted line.

The function IW | CIF(t) | $-f_1$ changes monothonically from the initial value to the final frequency. Similarly as for beating (cf. (3.1)), at the initial moment at that the signal amplitude of the frequency f_1 is maximal, the value IW | CIF(t) | $-f_1$ corresponds to the frequency f_1 (0 in the Fig. 2). At the moment of the amplitudes equalisation t_e the value IW | CIF(t) | $-f_1$ equals the average of the frequency change. After the time T_{60} measured from the moment of the frequency change, the value of the signal amplitude of frequency f_2 is considerably greater than that of frequency f_1. Then the value IW | CIF(t) | $-f_1$, similarly as for beating, equals the final frequency f_2.

The changes in the normalized IW | CIF(t) | function, obtained as a result of dividing the second component of the formula (3.6) by the value of the frequency change Δf, are shown in Fig. 3.

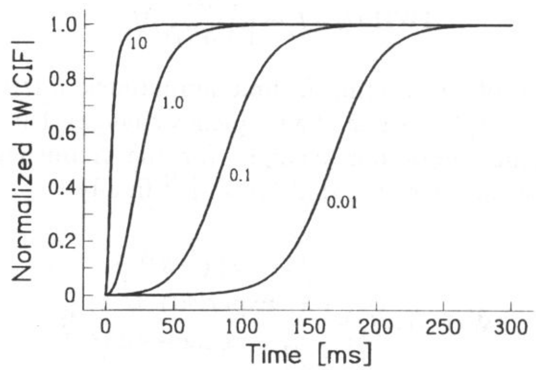

Fig. 3. Changes in the normalised IW | CIF(t) | function for selected values of the amplitude ratio δ ($T_{60}=0.5$ s).

The value of the amplitude ratio δ is a parameter of particular curves. For $\delta=10$ (the amplitude of the signal of final frequency is 10 times greater than of the initial frequency), changes in the function IW | CIF(t) | have a course similar to the frequency change for the signal sent into the room. For $\delta=0.01$, the time interval of changes in the function IW | CIF(t) | will be much longer. From 0 to about 100 ms these changes are small.

In Fig. 4 and 5 changes in the normalised curves IW | CIF(t) | for selected values of the room reverberation time are shown however Fig. 4 referes to the constant value $\delta=0.1$ whilst Fig. 5 refers to $\delta=1$.

For small values of the reverberation time $T_{60}=0.1$ s, the IW | CIF(t) | changes occur almost immediately. The time interval of the transition from the initial to the final frequency is very short and almost independent from the value of the amplitude

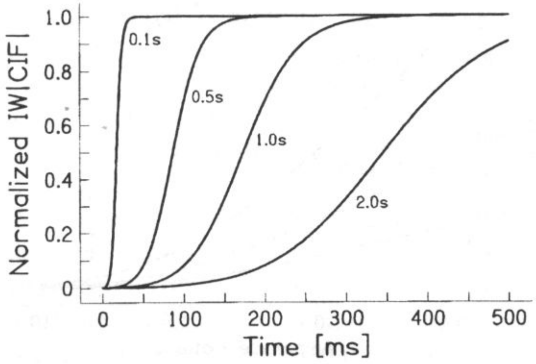

Fig. 4. Changes in the normalised IW | CIF(t) | function for selected reverberatin time values ($\delta=0.1$).

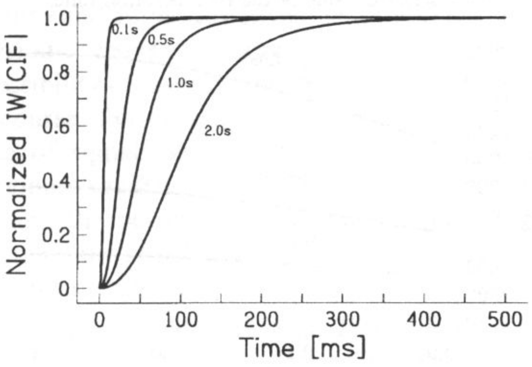

Fig. 5. Changes in the normalised IW | CIF(t) | function for selected reverberation time values ($\delta=1$).

ratio. An increase of the reverberation time causes an increase in that interval. The rate of the IW | CIF(t) | changes is greater for large amplitude ratios δ, but it decreases with the increase in the reverberation time.

As was mentioned earlier, in the initial phase of the magnitude of the instantaneous frequency changes, i.e. up the moment t_e, these changes appear in the proximity of the initial frequency f_1. At moments later than t_e, oscillations appear close to the final frequency f_2. The final frequency is reached after a time equal to the room reverberation time. In Fig. 6 the relation of t_e versus the amplitude ratio δ is shown. A similar relation for the time $T_{60}-t_e$ is shown in the Fig. 7. These relations were calculated from Eq. (2.8). The value of the reverberation time T_{60} is a parameter of the particular curves.

The increase of the amplitude ratio causes a considerable decrease of the time interval of oscillations around the initial frequency Fig. 6. The value of that interval increases proportionally to the room reverberation time. The time interval corresponding to the oscillations around the final frequency almost does not depend on the amplitude ratio δ (Fig. 7) and is much larger than similar values for oscillations around the initial frequency, especially for large values of δ. However, attention

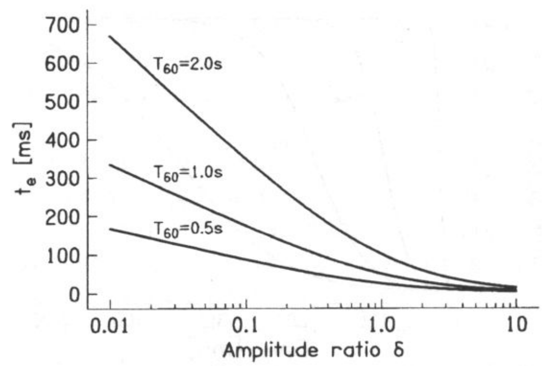

Fig. 6. The relation of the time interval of oscillations around the initial frequency versus amplitude ratio δ for selected value of the reverberation time.

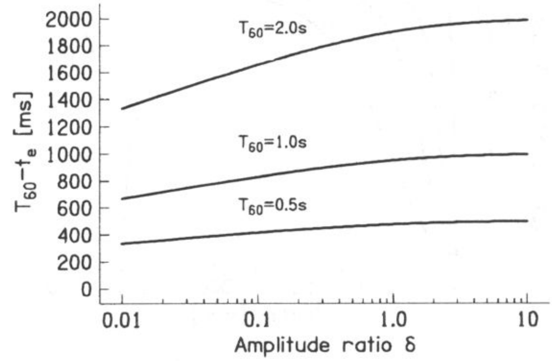

Fig. 7. The relation of the time interval of oscillations around the final frequency versus amplitude ratio for selected value of the reverberation time.

should be paid to the fact that the changes of IW $|\,\mathrm{CIF}(t)\,|$ in the final phase, i.e. for moments close to the value of the reverberation time, are small (cf. Fig. 3–5).

4. Discussion

The subject of the previous analysis was the physical aspects of the complex instantaneous frequency magnitude and envelope changes accompanying them as a result of the frequency change in a room. Basing on the modified IWAIF model, a measure (function) was found that allows to predict the possibility of pitch changes resulting from frequency change in a room. As opposed to elementary beating of nonsteady envelope and instantaneous frequency changes and settled average pitch (under assumptions described in Sec. 3.1) the calculated changes of pitch after a frequency change in a room have a nonsteady (transient) character. The continuous change of an amplitude ratio of spectral components with different but constant frequencies is the cause of nonsteady pitch changes predicted on the base of IWAIF model. The calculated predicted continuous pitch change exists in a room in a practically finite time interval. This interval can be very short but, especially in rooms with a large reverberation time, it can reach considerable values (cf. Fig. 6–7). The value of the steady state amplitude ratio of both considered signals also determines the duration of the calculated pitch change. The IWAIF model shows that, as a result of instantaneous frequency envelope weighting the oscillating instantaneous frequency changes are smoothed. Thus, the perceived pitch can be characterised by a number (beating) or by a function describing continuous pitch changes (frequency change in a room). From the point of view of room acoustics it is important to prove that the instantaneous frequency changes, registered in a room, are perceived by a listener. Predicted by the analogy to beating, the pitch changes in a room for the considered nonsteady signal allows to determines a possible sound features change in the room. This is important and useful because changes in the instantaneous frequency and amplitude envelope, similar to those discussed, also exist for a sound with periodic frequency changes.

The predicted possibility of transient pitch changes depends on a few factors such as the duration and course of instantaneous frequency changes. It is also related to the frequency difference of signals sent to the room determining the instantaneous frequency and envelope oscillation frequency. This problem requires separate experiments concerning the perception of predicted pitch changes. Especially the problem of averaging and the possibility to follow pitch changes must be examined in details. It is also important whether and to what extent the model used for beating sinusoids can be useful for the instantaneous pitch analysis. These investigations, exceeding the subject of this paper, have not been done so far.

But it is encouraging that a sudden sound frequency change causes effects perceived in a room, especially in a range of low acoustic frequencies. A frequency change in the range of several Hz gives a sensation similar to vibrato while playing music instruments. For higher acoustic frequencies and large values of the frequency change perceived sensation is similar to a metallic click. Such different sound effects are probably related to the frequency of envelope and instantaneous frequency changes depending on the frequency difference value. For small values of the frequency difference envelope changes are more clearly perceived than instantaneous frequency changes. It is difficult to relate the pitch perception as an envelope-weighted instantaneous frequency for the mentioned assumption. For larger values of the frequency change, the amplitude envelope as well as the instantaneous frequency change at a larger rate. This corresponds to the perception of an averaged or continuously changed pitch.

5. Conclusions

The analysis of instantaneous frequency and envelope changes (2.6), (2.7) occuring in a room as a result of the frequency change shows that those changes have a character of synchronously appearing envelope and instantaneous frequency changes. Therefore the result of these changes can be treated as a simultaneous amplitude and frequency modulation of finite duration. The synchronous appearing of both kinds of the modulation causes that the possibility of an instantaneous frequency change perception is strongly dependent on the amplitude envelope changes [5, 6]. The IWAIF model described in [1, 2] was used as an attempt to interpret the changes observed in a room from the perceptual point of view. These papers concerned mainly beating and their basic aim was rather the estimation of average pitch changes in time. The use of the IWAIF model for the amplitude ratio of two tones varying in time allowed to calculate "pitch values" in the following time intervals after the frequency change in a room. These calculated values of pitch are changing continuously in a time interval that equals the room reverberation time. One should be aware that proper mathematical calculations of pitch changes do not strictly correspond to the perceived pitch. It results not only from envelope and instantaneous frequency changes more complex than those for beating. Among others it seems to be important that these changes take a finite time interval and their rate depends on the value of the frequency change in the room. The other reason for some ambiguity in the interpretation of the changes perceived in a room is that it has not been clearly known so far what is the mechanism of pitch sensation. But generally it is known that a frequency change influences not only the signal pitch change but also it loudness [11] and an amplitude envelope change influences the change in the loudness and pitch [15].

Independently of the model we use to estimate frequency changes in a room, it can be generally said that simultaneous envelope and instantaneous frequency changes can be perceived as a transient (i.e. of finished duration) change of pitch.

Appendix
Complex instantaneous frequency

Let us consider a real signal resulting from the superposition of two tones with slow envelope changes (narrow band signal)

$$r(t)=x_1(t)\cos\omega_1 t+x_2(t)\cos\omega_2 t, \tag{A1}$$

where $x_1(t)$, $x_2(t)$ are functions describing the amplitude envelope change. To create an analytic signal let us calculate the Hilbert transform of the real signal (A1)

$$Hi\{r(t)\}=x_1(t)\sin\omega_1 t+x_2(t)\sin\omega_2 t. \tag{A2}$$

Making use of (A1) and (A2), an analytic signal corresponding the real signal $r(t)$ will be equal to

$$r_a(t)=r(t)+jHi\{r(t)\}. \tag{A3}$$

The envelope of the analytic signal is:

$$|r_a(t)| = \sqrt{[r(t)]^2 + [Hi\{r(t)\}]^2}. \tag{A4}$$

The analytic signal envelope is a real function that equals the envelope of the resultant real signal (A1). The phase of the analytic signal is:

$$\varphi_a(t) = \operatorname{arctg}\left[\frac{Hi\{r(t)\}}{r(t)}\right]. \tag{A5}$$

The instantaneous frequency (IF) is calculated as a time derivative of the analytic signal phase (A5)

$$\mathrm{IF}(t) = \frac{d\varphi_a(t)}{dt}. \tag{A6}$$

In practice phase changes are referred to a constant frequency value f_0 for which the phase is a linear function of time $\varphi(f_0) = 2\pi f_0 t$. Thus, the instantaneous frequency changes will be described as follows

$$\mathrm{IF}(t) = \frac{d\tilde{\varphi}_a(t)}{dt} + 2\pi f_0, \tag{A7}$$

where $\frac{d\tilde{\varphi}_a(t)}{dt}$ is a time varying component of the instantaneous frequency.

The resultant signal corresponding to the real signal (A1) is given by

$$r(t) = |r_a(t)| \cos\left[\int \mathrm{IF}(t)dt\right] = \mathrm{Re}\left\{\exp\left[\ln|r_a(t)| + j\int \mathrm{IF}(t)dt\right]\right\}. \tag{A8}$$

The expression

$$\mathrm{CI}\Phi(t) = \ln|r_a(t)| + j\int \mathrm{IF}(t)dt, \tag{A9}$$

is a complex phase of the resultant signal. The complex instantaneous frequency is

$$\mathrm{CIF}(t) = \frac{dCI\Phi(t)}{dt} = \frac{1}{|r_a(t)|}\frac{d|r_a(t)|}{dt} + jIF(t). \tag{A10}$$

The magnitude of the complex instantaneous frequency (the real function describing the frequency change) can be calculated from

$$|\mathrm{CIF}(t)| = \sqrt{\left[\frac{1}{|r_a(t)|}\frac{d|r_a(t)|}{dt}\right]^2 + [IF(t)]^2}. \tag{A11}$$

Let us notice that only for signals with a constant envelope the magnitude of the complex instantaneous frequency equals the instantaneous frequency IF(t).

For signals with simultaneous envelope and instantaneous frequency changes, the value of the complex instantaneous frequency magnitude depends on both the instantaneous frequency IF(t) and the real part of the complex instantaneous frequency changes.

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