

## PIEZOMAGNETIC PARAMETERS OF THE MAGNETOSTRICTIVE MATERIALS

Z. KACZKOWSKI

Polish Academy of Sciences  
Institute of Physics  
(02-668 Warszawa, Al. Lotników 32/46, Poland)

Piezomagnetic parameters of magnetostrictive materials and transducers are discussed. Properties of piezomagnetic materials are characterized by piezomagnetic coefficients occurring in the piezomagnetic equations and by the other physical parameters and quantities. A measure of the effectiveness of energy conversion is a magnetomechanical coupling coefficient, which, in addition to mechanical, magnetomechanical and magnetic quality factors, a piezomagnetic dynamics and an electroacoustical efficiency, a vibration amplitude, etc., permits to compare properties of the piezomagnetic materials and transducers. These parameters are useful in both theory and applications of piezomagnetic materials and transducers.

### 1. Introduction

Magnetomechanical and magnetoacoustical properties of the magnetic materials are very important in the theory and applications. Magnetomechanical, and mechanomagnetic phenomena in solids have a long, nearly four hundreds years, history, e.g. [1-17]. The physical properties of solid state are defined by relations between measurable quantities. In many cases, e.g. mechanical, thermal, electrical and magnetic properties are treated in isolation from others. Some connections between them, e.g. between electric ( $E$ ) and magnetic ( $H$ ) fields and magnetic inductions ( $B$ ) or electric displacement ( $D$ ), strains ( $S$ ), stresses ( $T$ ), heat or entropy per unit volume ( $S_e$ ) and temperature ( $Tem$ ) for the simplest interactions, thermodynamically reversible, are shown in Fig. 1. In a case of the nondirectional physical quantities, i.e. scalars (or tensors of zero rank), e.g. in temperature ( $Tem$ ), entropy ( $S_e$ ) or density ( $\rho$ ), their values are completely specified by giving single numbers. The quantities defined with reference to directions, called vectors (or tensors of the first rank), e.g. electrical field ( $E$ ) and displacement ( $D$ ), or versors, e.g. magnetic field ( $H$ ) and induction ( $B$ ), are defined by their magnitude and direction or by three mutually perpendicular components, which are simply projections of the vectors on the 3 perpendicular axes and are described by values with suffix 1, 2 or 3, or  $x$ ,  $y$  or  $z$ , corresponding respective axes, e.g.  $H_x$  or  $B_1$ . The higher ranks tensors, e.g. tensors of the second rank are described by nine components (written down in a square array, enclosed by square brackets) with two subscripts, [the first ( $i = 1, 2, 3$ ) gives the row and

the second ( $j = 1, 2, 3$  the column)], e.g. permittivity ( $\epsilon_{11}$ ), permeability ( $\mu_{13}$ ), stress ( $T_{ij}$ ), strain ( $S_{ij}$ ), where when  $i = j$ :  $\mu_{11}, \mu_{22}, \mu_{33}$  are the components of the leading diagonal and in the case of  $T_{ii}$  they are normal components of tensile stress and  $T_{12}, T_{21}, T_{13}, T_{31}, T_{23}$  and  $T_{32}$  are the shear components. All second-rank tensor properties are centrosymmetrical. If the stresses are applied to certain body they will develop an electric moment (piezoelectricity) or change of magnetic induction (magnetoelasticity), whose magnitudes are proportional to the applied stresses. The proportionality coefficients are piezoelectric constants ( $d_{ijk}$ ) or piezomagnetic (stress) sensitivity (coefficients) ( $d_{ijk}$ ) and other piezomagnetic coefficients (or piezoelectric constants) ( $e_{ijk}, g_{ijk}$  and  $h_{ijk}$ ) and they are third-rank tensors with 27 components. The (elastic) compliances  $s_{ijkl}$  (or elasticity moduli  $E_{ijkl}$  or Young's moduli  $Y_{ijkl}$ ) and the (elastic) stiffnesses  $c_{ijkl}$  (or elastic constants) are components of fourth-rank tensors. The number of subscripts equals the rank of the tensor. Piezoelectric phenomena have an analogy in a piezomagnetism. Piezomagnetic parameters of magnetostrictive materials are discussed. The effects in small signal range are linear and relationships are reversible.

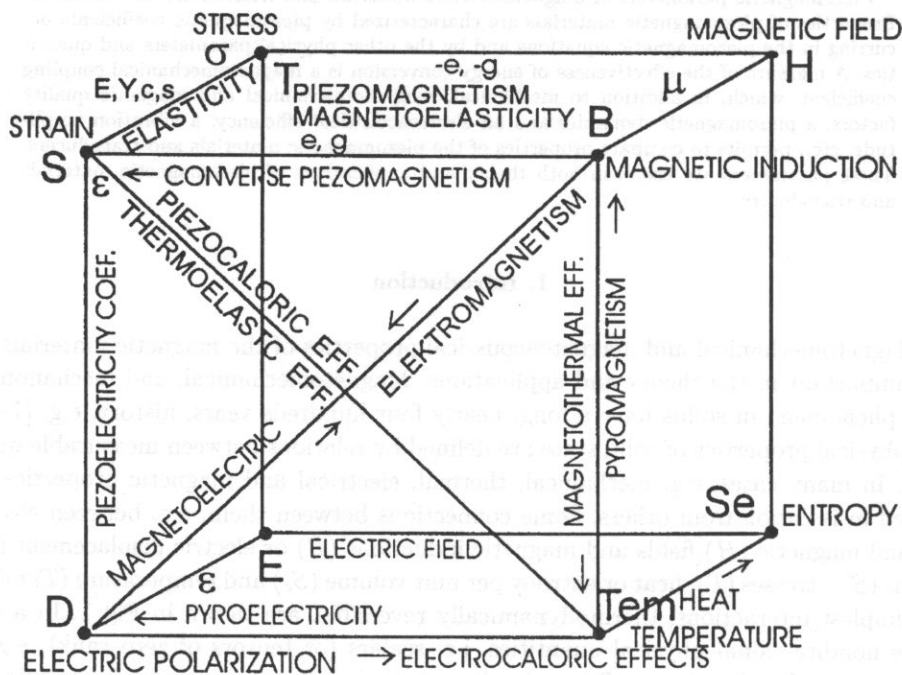


Fig. 1. The relations between the electrical, mechanical, magnetic and thermal properties.

All types of sound have as their source some mechanically vibrating body.

These vibrations may be produced or affected by magnetic field and, conversely, the sound vibrations may effects on magnetic properties (magnetoacoustics).

The dimensions, elasticity moduli, sound velocities and also electrical resistivity, are less or more changed in the applied magnetic field.

## 2. Brief history

In 1837 C.G. PAGE at New Haven heard clicks when magnetic materials were either magnetized or demagnetized [2]. Strictly he investigated coil of 40 turns, placed vertically between the poles of a hors-shoe magnet, and found that it was produced a distinct tone in the magnet when electric current was started or stopped in the coil [2-5]. Similarly, it has been observed that when electromagnet was rotated between the poles of magnet a distinct sound was heard as from a tuning-fork. If magnetization changes are rapid the resulting mechanical transients generate a sound or ultrasound waves, e.g. the humming noises from transformer.

Some years later James Prescott Joule discovered that an iron bar placed magnetic field increased its length. His results JOULE presented on February 16, 1842 in a lecture delivered at the Victoria Gallery in Manchester [6, 7].

In 1844 WERTHEIM discovered that when a magnetic rod was immersed in a longitudinal magnetic field and twisted, its longitudinally directed magnetization vectors were changed to helical directions and a transient voltage was generated between its ends [8]. The inverse Wertheim effect occurs when a magnetic rod placed in an axial magnetic field twists when a voltage is applied to its ends. The circular magnetization is connected with the current flow after applying a voltage.

In 1846 GULEMIN observed that the bent iron rod tended to straighten when it was exposed to a longitudinal magnetic field [9].

In 1847 MATEUCCI observed that when a magnetic rod laying lengthwise in a magnetic field was twisted, then its magnetization changed [10]. He found that twisting and untwisting the iron cylinder (which ends were connected to a galvanometer) caused induced currents.

The Joule effect has its inverse in VILLARI effect, i.e. changes of magnetization produces by an external tension, discovered in 1865 [11].

Magnetostrictive materials exhibit either an increase or a decrease of magnetization with increasing strains, depending on the sign of the magnetostriction and the direction of the applied forces (Fig. 2).

A positive Villari effect is defined as an increase in magnetization ( $M$ ) or in permeability ( $\mu$ ), when the material with the positive magnetostriction (e.g. iron-rich metallic glasses, Alfers, Alcofers or Permendurs) is extended and a decrease of the  $M$  or  $\mu$  when the material is compressed.

A negative Villari effect means a decreasing of the magnetization or permeability during extension or an increase of  $M$  or  $\mu$  during compression in materials with  $\lambda < 0$ , e.g. in nickel and nickel ferrites.

In iron and some other iron-rich alloys, e.g. steels, the permeability increases with the strain in weak magnetic fields ( $H$ ) and decreases in higher fields  $H$ . Between these states is a point on the magnetization curve (Villari Reversal - VR in Fig. 2) where the magnetostriction is unaffected by the strain.

Villari discovered also that an iron bar lengthen when magnetized by a weak magnetic field but contracts when a particular field is exceeded. So Villari effect also mean the change of the magnetostriction sign in iron (Villari's point) [11].

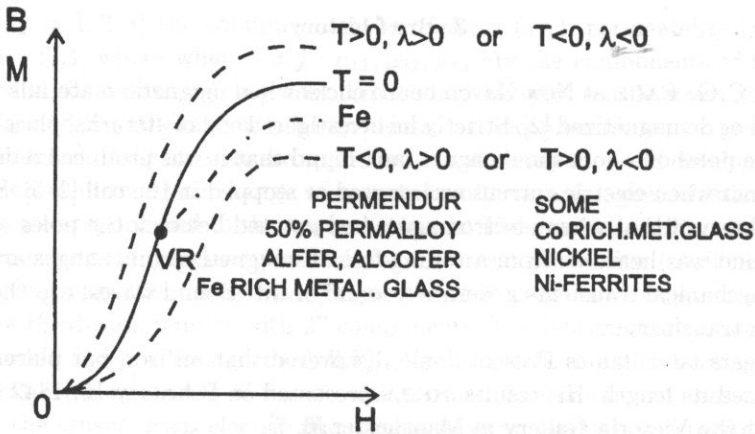


Fig. 2. Villari effect for iron (Fe), materials without stress ( $T = 0$ ), stretched ( $T > 0$ ) and compressed materials ( $T < 0$ ) with positive or negative magnetostriction ( $\lambda$ ).

In transverse Villari effect is a change in the magnetization (or permeability) that occurs in a direction transverse to that of the mechanical strain [11].

In WIEDEMANN effect a wire, subjected to an axial field and to a circular field produced by the current flowing through this wire, twists [12]. If a magnetic rod, tube or wire are twisted while the electric current  $I$  is passing through them the samples become axially magnetized (inverse Wiedemann effect). Both of this effect are a result of imposing linear magnetization upon the circular magnetization that accompanies a longitudinal flowing electric current. A voltage ( $U$ ) will be generated in a coil surrounding a wire (or rod) while it is being twisted. In the presence of torsional strains, the alignments of some of the magnetization vectors are diverted from the predominantly circular directions to a helical direction [12]. When the direct current passes through the wire and one its end is fixed the other free end twists. If one magnetic field (circular or longitudinal) is alternating current field then in a magnetostrictive core occur the torsional vibrations.

A change in magnetization of a magnetostrictive material subjected to hydrostatic pressure is known as the NAGAOKA-HONDA effect [13]. It is inverse effect to the BARRETT effect, i.e. volume magnetostriction, discovered in 1882 [14].

In 1894 ROZING discovered a hysteresis of the magnetostriction as a function of magnetic field [15].

In 1897 GUILLAUME found that face centered cubic  $Fe_{65}Ni_{35}$  alloy shows a very small (invariant) thermal expansion coefficient ( $< 1.2 \times 10^{-6}$ ) at room temperature [16]. The low thermal expansion of Invar alloys is due to the large spontaneous volume magnetostriction.

HONDA, SHIMIZU and KUSAKABE discovered  $\Delta E$ -effect, i.e. changes of the elasticity moduli with magnetic polarization [17].

W. VOIGT pointed out that a true piezomagnetic effect is theoretically possible in 29 of the 32 crystal classes [18, 19].

HONDA and SHIMIZU investigated effect of various frequencies (up to 150 Hz) on the longitudinal frequencies [20] and PIERCE increased this range to the radiofrequencies [21].

In 1919 BARKHAUSEN discovered jumps, i.e. irregularities in the magnetization during the changes of the magnetization [22]. It is generally accepted that lattice imperfections, e.g. dislocations, pores, vacancies, grain boundaries, i.e. all pinning sites impede the domain wall motion, result in Barkhausen jumps on the order of magnitude of 10 nT.

In 1920 GUILAUME found that in  $\text{Fe}_{55}\text{Ni}_{45}$  alloys the elastic properties are insensitive to temperature (elasticity invariable – elinvar) around room temperature [23]. The better Elinvar type alloys contain also Cr, e.g.  $\text{Fe}_{52}\text{Ni}_{36}\text{Cr}_{12}$  or Fe-Ni-Mo, Ti and Be.

WEBSTER had investigated a magnetostriction ( $\lambda$ ) in single crystals of iron and nickel and found the positive magnetostriction for [100] direction in iron, negative ( $\lambda$ ) in nickel for all directions and for [111] direction in Fe and a change of sign for [110] direction in iron [24].

When a magnetostrictive material is combined in a sandwich with a piezoelectric material (in which output voltage is connected with the change of dimensions) and mechanical coupling between these two materials is maximized, the voltage across the piezoelectric material is proportional to the magnetic field and this phenomenon is known as a magnetostrictive-piezoelectric effect, e.g. [25].

If an electromagnetic wave traveling in the waveguide stimulates Larmor precession in ferrite, the presence of that wave can be detected in piezoelectric transducer by the observed voltage, e.g. [25].

In 1957 DZIALOSHINSKII from the consideration of magnetic symmetry, drew attention to several antiferromagnetic materials in which piezomagnetic effect should be observed [26]. The first observation of natural piezomagnetism of antiferromagnetics was made by BOROVIK-ROMANOV in  $\text{MnF}_2$  and  $\text{CoF}_2$  [27]. The reverse piezomagnetic effect was observed by ANDERSON, BIRSS and SCOTT in hematite [28].

In the GARSHELIS effect, the magnetization of a wire spring fashioned from a magnetostrictive material immersed in a constant magnetic field is a function of coil spring deflection [29]. If the magnetization is a result of a direct current flowing through the spring, the Garshelis effect becomes inverse Wiedemann effect and if a remanent magnetization is directed along the wire axis, the Garshelis effect becomes Wertheim effect, e.g. [25].

Abrupt domain wall movements after depinning or closure domain collapse or formation, which is the reason of Barkhausen jumps, also generate transient elastic stress waves due to changes in magnetostrictive strain produced in the volume swept by the domain walls. These elastic waves can be detected by a piezoelectric transducer bonded to the specimen and are known as a magnetoacoustic emission [30]. The Barkhausen effect and the magnetoacoustic emission both arise from micromagnetic changes. Such changes are affected by the local magnetostriction and internal stresses. Consequently both signals are potentially sensitive to the stress level, because stress will influence the energetically favored domain configurations.

### 3. Magnetostriction

Magnetostriction embraces the phenomena involving the interaction between magnetization and mechanical stresses and strains and refers both to the changes in di-

mensions (linear magnetostriction, longitudinal and transverse magnetostriction, JOULE effect [6, 7], volume magnetostriction or BARRETT effect [14]) and to the changes in elastic properties ( $\Delta E$  effect [17]) that occur in magnetic materials in the presence of imposed magnetic fields and to the magnetization changes (magnetoelasticity, VILLARI effect [11]) that occur in magnetic materials exposed to mechanical stresses, e.g. [31–40].

The linear longitudinal and transverse relative changes of dimensions, i.e. length ( $l$ ), width ( $w$ , or diameter  $d$  or radius  $r$ ) or thickness ( $t$ ), in the applied magnetic field ( $H$ ) are known as a linear longitudinal ( $\lambda = \lambda_l = \lambda_{\parallel} = \Delta l/l$ ), or transverse ( $\lambda_t = \lambda_{\perp} = \Delta t/t$ , or  $\Delta w/w$ , or  $\Delta r/r$ ) magnetostrictions (or longitudinal or transverse Joule effects). A linear longitudinal magnetostriction, i.e. change of length or perpendicular dimensions, is the anisotropic strain that accompanies a change in the value and direction of magnetization (Fig. 3).

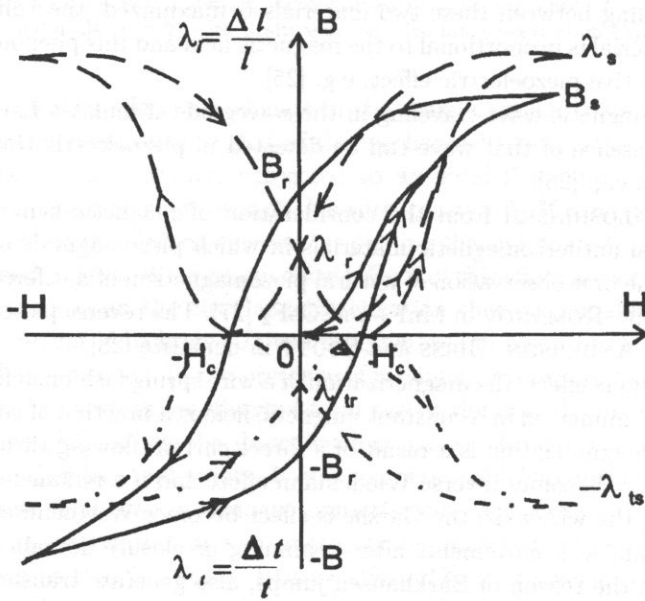


Fig. 3. Initial curve and hysteresis loop of the magnetic induction ( $B$ ) and initial curve and butterfly loops of the linear magnetostriction ( $\lambda$ ). Even effect of the longitudinal ( $\lambda_l = \lambda_{\parallel}$ ) and transversal ( $\lambda_t = \lambda_{\perp}$ ) magnetostrictions is observed, i.e.  $\lambda_l(+H) = \lambda_l(-H)$  and  $\lambda_t(+H) = \lambda_t(-H)$ .  $B_S$  (saturation induction),  $\lambda_s$  and  $\lambda_{ts}$  (saturation magnetostrictions for longitudinal or transversal directions) are the values at the magnetic saturation ( $H_s$ ) and  $B_r$ ,  $\lambda_r$  and  $\lambda_{tr}$  at the magnetic remanence.

In many materials, e.g. in nickel or in some of metallic glasses, the changes in the length measured at right angles to the applied field are about half of the longitudinal magnetostriction and opposite in sign, i.e.  $\lambda_t = -(1/2)\lambda_l$  [31–41].

Magnetostriction may have a positive (elongation) or negative (contraction) sign. When the magnetization ( $M$ ) approaches the saturation ( $M_s$ ) or saturation induction ( $B_s$ ), the magnetostriction approaches its limiting value, i.e. a saturation magnetostric-

tion ( $\lambda_s$  in Fig. 3), which belongs to the fundamental parameters of the magnetic materials, e.g. [31–42].

The magnetostriction is an even function of the magnetic field, i.e. a change of the sign or a reverse of the direction of applied magnetic field does not change the sign of the magnetostriction (Fig. 3). With the previous discussed fundamental magnetomechanical effects, as magnetostriction,  $\Delta E$  effect and magnetoelasticity (Villari effect) there are associated other magnetomechanical phenomena, e.g. PAGE effect [2], WERTHEIM effects [8], GUILLEMIN effect [9], MATEUCCI effects [10], WIEDEMANN effects [12], NAGAOKA–HONDA effect [13], GARSHELIS effect [29], magnetostrictive-piezoelectric effect [25], magnetostrictive detection effect [25], BARKHAUSEN effect [22], magnetoacoustic emission [30],  $\Delta G$ ,  $\Delta K$  and  $\Delta c$  effects, Invar effect [16], Elinvar effect [23] and piezomagnetic effect [18, 19, 26–28].

Magnetostriction usually decreases in magnitude with increasing temperature. In the most of magnetic materials the saturation magnetostriction is proportional to ca. squared magnetization, i.e.  $\lambda_s \sim M_s^2$ . Magnetostriction may be increased or decreased by stress depending on its sign and the direction of the acting forces. When  $\lambda > 0$ , tension usually decreases  $\lambda$ , and when  $\lambda < 0$ , tension increases a absolute magnitude of  $\lambda$  making it more negative, e.g. [34–40].

In strong magnetic fields the linear magnetostriction approaches a limiting value, superposed on which is a relative small length change. Then the material expands or contracts equally in all directions and its volume is changing (volume magnetostriction or Barrett effect ( $\omega = \Delta V/V = \Delta l/l + \Delta t/t + \Delta w/w$ )) [31–42].

All reported magnetostrictive phenomena are connected with magnetic structure [31–40].

The changes of the dimensions and other magnetostrictive phenomena depend on non-180° Bloch-wall movements and on domain vector rotations.

Many magnetic, mechanical and ultrasonic properties, e.g. permeability, coercivity, remanent magnetization, losses, elasticity moduli and ultrasound velocities depend very critically on the fundamental magnetostrictive properties.

#### 4. Piezomagnetism

When the magnetostrictive materials or transducers are placed in an alternating magnetic field the mechanical vibrations occur because of the dimension changes (Fig. 4). If the frequency of ac field is equal to the mechanical resonance frequency of the polarized core, the amplitude of mechanical vibrations will be multiplied by mechanical quality factor  $Q_m$ .

The piezomagnetic samples are polarized by a steady (static) magnetic field upon which the alternating magnetic field is superposed. The resulting state is an increase of magnetization when ac field is in one direction with static one and a decrease of magnetization when ac field is in the reverse direction. These two states in materials with positive magnetostriction are accompanied by an increase of length and a decrease of length (Figs. 3 and 4), respectively, e.g. in Permendurs, Permalloys, Alfere and Alcofers,

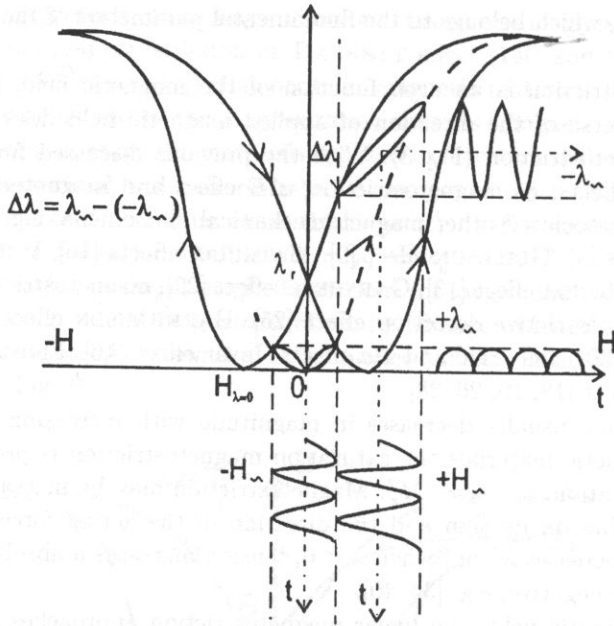


Fig. 4. Magnetostriction and piezomagnetic effects in the polarized by magnetic bias field ( $H$ ) magnetostrictive materials or at remanence, i.e.  $[\lambda_-(+H_-) = -\lambda_-(-H_-)]$  at constant (static)  $\lambda_-$  or at  $\lambda_r$ . For  $\lambda_- = 0$ ,  $\lambda_- (+H_-) = \lambda_- (-H_-)$ .

some rare-earth elements, compound and alloys and iron-rich metallic glasses [31–45]. In materials with negative magnetostriction, e.g. in nickel and nickel-ferrites and some rare-earth elements, compound and alloys in crystalline or amorphous state [31–45], or in materials with positive magnetostriction but for transverse directions (Fig. 3), the increase of magnetization is accompanied by the decrease of length and with the decrease of magnetization is connected the increase of length.

When the variations in mechanical and magnetic parameters are small compared with the initial values of these parameters, and they are reversible, the magnetomechanical interactions can be represented as effectively linear phenomena [43–50].

Piezoelectric phenomena [50–55] have an analogy in a piezomagnetism [18, 19, 26–28, 43–52].

The piezoelectricity is an electricity developed in certain crystals with non-center of symmetry by mechanical strains and it is also the effect of an electric field in producing in such crystals an expansion along one axis and a contraction along another. The effects are linear and relationships are reversible [43–57].

An effect phenomenologically equivalent to natural piezomagnetism, i.e. a proportionality between magnetization and stress, is observed in biased magnetostrictive materials (Fig. 4) [43–51].

This forced piezomagnetism is observed in all magnetostrictive materials when they are polarized by the bias magnetic field or at the magnetic remanence (Fig. 4).



5. Piezomagnetic equations

The piezomagnetic interactions are represented by the top face of the cube, presented in Fig. 1. There are four variables: two mechanical, i.e. stress ( $T$  or  $\sigma$ ) and strain ( $S$  or  $\varepsilon$  or  $\lambda$ ), and two magnetic, i.e. magnetic field ( $H$ ) and magnetic induction ( $B$ ) or magnetization ( $M$ ) or magnetic polarization ( $J = \mu_0 M$ ) [43–54, 56–62].

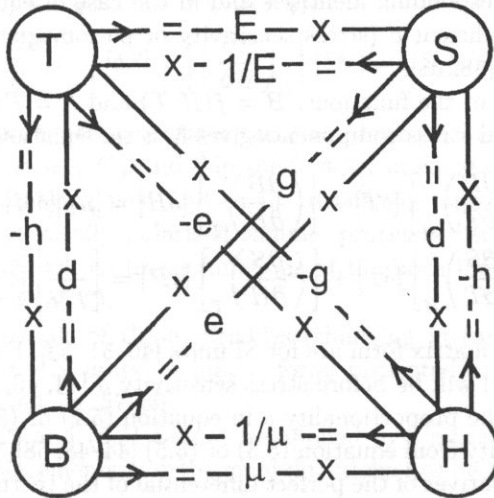


Fig. 5. The mechanical ( $E$ ), magnetic ( $\mu$ ) and piezomagnetic ( $d, e, g, h$ ) relations between mechanical ( $T$  and  $S$ ) and magnetic ( $B$  and  $H$ ) properties.

Starting points of the piezomagnetic (or magnetostrictive) equations are: a relation for the magnetization curve ( $B = \mu H$ ) and Hooke’s Law ( $T = ES$ ) to which the terms representing magnetomechanical interaction should be added (Fig. 5), i.e.

$$B = \mu_0(H + M) + \Delta B = \mu_0 H + J + \Delta B, \tag{5.1}$$

$$B = \mu_0 \mu' T H + d T = \mu_T H + d T, \tag{5.2}$$

where  $M$  is the magnetization,  $J$  – the magnetic polarization,  $\mu' = \mu/\mu_0$  is the relative value of the permeability  $\mu$ , and  $\mu_0 = 4\pi 10^{-7} H/m$  is magnetic constant (defined in an old MKSA system as the vacuum permeability), the increment  $\Delta B$  (positive or negative according to the sign of the magnetostriction and of the direction of the acting force) represents a change of the induction caused by the magnetomechanical interaction, e.g. by stress  $T$ , and coefficient of the proportionality  $d$  is the piezomagnetic (or stress) sensitivity [43–54, 57–59], and

$$S = \frac{T}{E} + \Delta S = \frac{T}{E_H} + d H, \tag{5.3}$$

where increment  $\Delta S$  represents a change of the strain  $S$ , and  $E_H$  is the modulus of elasticity at constant magnetic field ( $H$ ), connected with the magnetomechanical interactions caused by the changes of the  $H$ .

If the thermal terms are omitted, the application of suitable magnetic bias field ( $H_-$ ) and resulting mechanical bias magnetostriction ( $\lambda_-$ ) allows the use of four possible simple sets of the linear equations of state between four harmonic variables: the magnetic variables  $H$  and  $B$  (or  $M$  or  $J$ ) and mechanical variables  $S$  ( $= \sigma$ ) and  $T$  ( $= \varepsilon$ ), whilst one magnetic and one elastic variables are chosen as independent variables [43–54, 57–59]. Using this assumptions and taking into a consideration thermodynamic relations, it is possible to derive corresponding identities and in the case of equations (5.2) and (5.3) one piezomagnetic coefficient  $d$  (stress sensitivity or piezomagnetic sensitivity) will be in the both equations [48, 58].

The differentiation of the functions:  $B = f(H, T)$  and  $S = F(H, T)$  with respect to each magnetic field and stress components gives first set equations:

$$[\delta B] = \left[ \left( \frac{\partial B}{\partial T} \right)_H \right] [\delta T] + \left[ \left( \frac{\partial B}{\partial H} \right)_T \right] [\delta H] = [\mu_T][\delta H] + [d][\delta T], \quad (5.4)$$

$$[\delta S] = \left[ \left( \frac{\partial S}{\partial T} \right)_H \right] [\delta T] + \left[ \left( \frac{\partial S}{\partial H} \right)_T \right] [\delta H] = \left[ \frac{1}{E_H} \right] [\delta T] + [d][\delta H]. \quad (5.5)$$

These equations in matrix form are for SI units [46–51, 58]. For old system units the coefficient  $4\pi$  or  $(1/4\pi)$  will be before stress sensitivity  $d$  [44, 46, 48].

The coefficient of the proportionality  $d$  in equation (5.2) or (5.4) is identical with the piezomagnetic sensitivity from equation (5.3) or (5.5) [44–48, 58], because the second order mixed partial derivatives of the perfect differential of the thermodynamical potential  $\varphi$  are equal each other, e.g. [44, 48, 58],

$$\frac{\partial^2 \varphi}{\partial T_i \partial H_m} = - \left( \frac{\partial S_i}{\partial H_m} \right)_{T_i} = -d, \quad (5.6)$$

$$\frac{\partial^2 \varphi}{\partial T_i \partial H_m} = - \left( \frac{\partial M_m}{\partial T_i} \right)_{H_m} = - \left( \frac{\partial B_m}{\partial T_i} \right)_{H_m} = -d, \quad (5.7)$$

where  $i = 1, \dots, 6$ , and  $m = 1, 2, 3$  [44, 48, 53, 54, 58]. Naturally, in such case one should state precisely definitions of the magnetic and mechanical coefficients and thus the permeability should be determined at constant stresses  $\mu_T$ , i.e. permeability of the free vibrating sample, and the elasticity moduli  $E_H$  at constant magnetic field  $H$ , what corresponds an electrical source with internal impedance going to infinity ( $Z_i \rightarrow \infty$ ), i.e. at constant current  $I$  to which is proportional magnetic field (i.e.  $H = nI/l$ , where  $n$  is the number of turns of the coil and  $l$  is the length of magnetic path in coil), e.g. [44, 48]. The total differentials of the magnetic and mechanical functions (5.2) and (5.3) are given in set of the equations (5.4) and (5.5).

If the independent and dependent variables  $B$ ,  $H$ ,  $T$ , and  $S$  will be changed, the next three sets of the piezomagnetic equations may be derived, e.g. [44–54, 57–59]:

$$[\delta H] = \left[ \left( \frac{\partial H}{\partial T} \right)_B \right] [\delta T] + \left[ \left( \frac{\partial H}{\partial B} \right)_T \right] [\delta B] = \left[ \frac{1}{\mu_T} \right] [\delta B] - [g][\delta T], \quad (5.8)$$

$$[\delta S] = \left[ \left( \frac{\partial S}{\partial T} \right)_B \right] [\delta T] + \left[ \left( \frac{\partial S}{\partial B} \right)_T \right] [\delta B] = \left[ \frac{1}{E_B} \right] [\delta T] + [g][\delta B], \quad (5.9)$$

$$[\delta H] = \left[ \left( \frac{\partial H}{\partial S} \right)_B \right] [\delta S] + \left[ \left( \frac{\partial H}{\partial B} \right)_S \right] [\delta B] = \left[ \frac{1}{\mu_S} \right] [\delta B] - [h][\delta S], \quad (5.10)$$

$$[\delta T] = \left[ \left( \frac{\partial T}{\partial S} \right)_B \right] [\delta S] + \left[ \left( \frac{\partial T}{\partial B} \right)_S \right] [\delta B] = [E_B][\delta S] - [h][\delta B], \quad (5.11)$$

$$[\delta B] = \left[ \left( \frac{\partial B}{\partial S} \right)_H \right] [\delta S] + \left[ \left( \frac{\partial B}{\partial H} \right)_S \right] [\delta H] = [\mu_S][\delta H] + [e][\delta S], \quad (5.12)$$

$$[\delta T] = \left[ \left( \frac{\partial T}{\partial S} \right)_H \right] [\delta S] + \left[ \left( \frac{\partial T}{\partial H} \right)_S \right] [\delta H] = [E_H][\delta S] - [e][\delta H]. \quad (5.13)$$

In each set of piezomagnetic equations there are three piezomagnetic or magnetomechanical coefficients, e.g.  $\mu_T$ ,  $E_H$  and  $d$  in the first set in equations (5.4) and (5.5).

When a small alternating field and small alternating tensile stress are applied to a freely vibrating magnetically polarized sample, processes become reversible, the induction  $B$  and strain  $S$  will be linear functions of the both independent variables, i.e.  $B = f(H, T)$  and  $S = F(H, T)$ .

The perfect differentials of these variables (changing in each set the independent variables) will have in the SI units the matrix forms presented in the equations (5.4) and (5.5) and (5.8)–(5.13) [44–54, 57–60].

The equations (5.2) and (5.3) cannot be applied to a more general problem, since coupling between displacements in orthogonal directions has been completely neglected, e.g. [54].

For the general case, the coefficients in equations (5.2) and (5.3) are the matrix quantities rather than the scalar quantities, e.g. [43–54, 57–60]. In the matrix form these equations (5.2) and (5.3) are presented in the first set of piezomagnetic equations (5.4) and (5.5).

The piezomagnetic equations are valid for the adiabatic changes [43–54].

The elements of each of the matrices are determined, e.g. for equations (5.2) and (5.3), by expanding the induction and strain components in terms of all and magnetic field components. There are only six independent components of stress because for presupposing rotational equilibrium for elementary volume,  $T_{ij} = T_{ji}$  ( $i, j = 1, 2, 3$ ), and these components are arranged to form of the single-column matrix, where  $T_1 = T_{11}$ ,  $T_2 = T_{22}$ ,  $T_3 = T_{33}$ ,  $T_4 = T_{23} = T_{32}$ ,  $T_5 = T_{13} = T_{31}$  and  $T_6 = T_{12} = T_{21}$ .

The strain matrix is also represented as a six-components single-column matrix, where shearing strains  $S_4 = 2S_{23}$ ,  $S_5 = 2S_{13}$  and  $S_6 = 2S_{12}$ .

Since there are only three-components of axial magnetic vectors (versors),  $H$  and  $B$  (or  $J$  or  $M$ ) are written as three-components single-column matrices.

Elasticity moduli matrices at constant magnetic field  $E_{Hij}$  and at constant magnetic induction  $E_{Bij}$  will create  $6 \times 6$  matrices, in which  $E_{ij} = E_{ji}$ , because of the reciprocity [43–54, 57–60]. For one dimension case  $E_{H33}$  is the Young's modulus ( $Y_H$  or  $E_H$ ) and equation (5.3) is then valid.

Magnetic permeabilities matrices at constant stress (free vibrating sample)  $\mu_{Tij}$  and at the constant strain (clamped sample)  $\mu_{Sij}$  are  $3 \times 3$  array matrices, in which (it may be often assumed)  $\mu_{ij} = \mu_{ji}$ , e.g. [43–54].

The piezomagnetic coefficients relating the magnetic and mechanical properties, create  $3 \times 6$  or transposed  $6 \times 3$  matrices, i.e. the rows and columns in the second equations of the each set are interchanged [43–54].

The magnetized magnetics have the symmetry  $\infty/m$ , with an infinitesimal symmetry axis and symmetry plane normal to it: the combination engenders a center of symmetry. This is compatible with piezomagnetism, since the center of symmetry does not reverse the sign of an axial vector [18, 19, 54]. For the symmetry of  $6/m$  VOIGT [19] lists following piezomagnetic sensitivity matrix:

$$\begin{array}{cccccc} 0 & 0 & 0 & d_{14} & d_{15} & 0 \\ 0 & 0 & 0 & d_{15} & -d_{14} & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{array}$$

In each set of piezomagnetic equations there are three magnetomechanical coefficients, e.g.  $\mu_T$ ,  $E_H$  and  $d$  in the equations (5.2) and (5.3) or (5.4) and (5.5).

The partial derivatives evaluated at the bias values, one magnetic and one mechanical and one magnetomechanical (piezomagnetic) are the linear parameters, which are functions of these bias quantities but they are constant for given operation points for small signal analysis. The differentials in equations (5.4), (5.5), (5.8)–(5.13) may be then replaced by their small signal values, i.e. ac vector quantities of the magnetic field ( $H$ ) and magnetic induction ( $B$ ) [or magnetic polarization ( $J$ ) or ( $I$ ), or magnetization ( $M$ )] and two second-order tensor quantities: dynamical stress ( $T$  or  $\sigma$ ) and strain ( $S$  or  $\varepsilon$ ).

The choice of the independent variables is arbitrary, e.g.  $H = f(S, B)$  and  $T = F(S, B)$  [43–54, 57–60].

The piezomagnetic coefficients:  $d$ ,  $e$ ,  $g$ ,  $h$ , denoted respectively to piezoelectric constants [52–55], and the permeabilities  $\mu_T$  of the free vibrating sample (at constant stresses  $T$ ) and  $\mu_S$  of the clamped sample (at constant strain  $S$ ), the elasticity moduli  $E_H$  at constant magnetic field ( $H$ ), what corresponds an electrical source with internal impedance going to infinity ( $Z_i \rightarrow \infty$ ), i.e. at constant current, and  $E_B$  at constant magnetic induction ( $B$ ), what corresponds an electrical short circuited source, i.e. with impedance going to zero ( $Z_i \rightarrow 0$ ), and the magnetomechanical coupling coefficient ( $k$ ) are variables dependent not only on the temperature, stress, pressure but also on the ac and dc magnetic fields, e.g. [31–54, 57–60].

Dynamical properties of the magnetostrictive materials (and transducers) are characterized by their piezomagnetic (magnetomechanical) coefficients occurring not only in piezomagnetic equations (5.2)–(5.5) and (5.8)–(5.13) but also in other versions of these equations and units and by other coefficients [34, 36, 39, 40, 43–49, 52, 53, 56].

## 6. Magnetomechanical coupling coefficient and piezomagnetic dynamics

A measure of the effectiveness of energy conversion is a magnetomechanical coupling coefficient ( $k$ ), e.g. [49], which, in addition to mechanical ( $Q$ ) and magnetic ( $Q_\mu$ ) quality factors and an electroacoustical efficiency ( $\eta_{ea}$ ), a vibration amplitude, etc., permits to compare properties of the piezomagnetic materials and transducers, e.g. [43–54, 57–60].

Its analogue in piezoelectric materials is the coefficient of the electromechanical coupling [52-55].

A part of the energy ( $\Delta W$ ) supplied to the transmitting transducer is converted to energy of elastic oscillation ( $W$ ) and radiated into medium loading the transducer. The opposite process occurs in receiving transducer. The measure of this transformation is a squared coefficient of magnetomechanical coupling ( $k^2$ ).  $k^2$  defines, which part of the magnetic (or mechanical) energy ( $W$ ) is converted into the mechanical (or magnetic) energy ( $\Delta W$ ), e.g. [43-54], i.e.

$$k^2 = \frac{\Delta W}{W} = \frac{\frac{1}{2}B_T H - \frac{1}{2}B_S H}{\frac{1}{2}B_T H} = 1 - \frac{\mu_S}{\mu_T}, \quad (6.1)$$

where the magnetic induction  $B = \mu H$ , and  $\mu$  is the magnetic permeability of the freely vibrating sample (at constant mechanical stresses  $T$ ), i.e. for  $B_T$  is  $\mu_T$ , or of the clamped sample (at constant strains  $S$ ), i.e. for  $B_S$  is  $\mu_S$ , and  $H$  is magnetic field. In a receiver the mechanical energy is partly converted to the magnetic energy,

$$k^2 = \frac{\frac{1}{2}S_H T - \frac{1}{2}S_B T}{\frac{1}{2}S_H T} = \frac{\frac{1}{E_H} - \frac{1}{E_B}}{\frac{1}{E_H}} = 1 - \frac{E_H}{E_B} \approx a \frac{f_a^2 - f_r^2}{f_a^2}, \quad (6.2)$$

where stresses  $T = \sigma = ES = E\varepsilon$ , and  $S$  and  $E$  are strains and moduli of elasticity at constant magnetic field ( $S_H$  and  $E_H = 4l^2 c_H^2 = 4l^2 f_H^2 \rho \approx 4l^2 f_r^2 \rho$  for rods and  $E_H = \pi^2 d^2 c_H^2 = \pi^2 d^2 f_H^2 \rho \approx \pi^2 d^2 f_r^2 \rho$  for toroids) or at constant magnetic induction ( $S_B$  and  $E_B = 4l^2 c_B^2 = 4l^2 f_B^2 \rho \approx 4l^2 f_a^2 \rho$  for rods and  $E_B = \pi^2 d^2 c_B^2 = \pi^2 d^2 f_B^2 \rho \approx \pi^2 d^2 f_a^2 \rho$  for toroids), respectively [because ultrasound velocities  $c = (E/\rho)^{1/2}$ ] and  $\rho$  is the vibrator density.  $l$  is the length of the half-wave longitudinally vibrating rod and  $d$  is mean diameter of the radial vibrating resonator.  $f_r$  and  $f_a$  are the resonant (i.e. at maximum impedance) and antiresonant (i.e. at minimum impedance) frequencies and in low losses materials are equal to  $f_H$  and  $f_B$ , respectively, and  $a$  is shape coefficient (for half-wave transducer  $a = \pi^2/8$  and for toroidal resonator  $a = 1$ ) [43-49].

The magnetomechanical coupling coefficient is connected with the piezomagnetic coefficients occurring in piezomagnetic equations by the following dependencies [43-54, 57-60]:

$$k^2 = d^2 \frac{E_H}{\mu_T} = h^2 \frac{\mu_S}{E_B} = \frac{g^2 E_B \mu_T}{1 + g^2 E_B \mu_T} = \frac{e^2}{e^2 + E_H \mu_S} = 1 - \frac{\mu_S}{\mu_T} = a \left( 1 - \frac{E_H}{E_B} \right). \quad (6.3)$$

Generally, owing to internal mechanical losses, the complex values should be attributed to the magnetomechanical coupling and to the moduli of elasticity (or to the ultrasound velocities), where e.g. an internal friction coefficient ( $Q^{-1}$ ), or mechanical quality factor  $Q$  are calculated from the real and imaginary components of the dynamical modulus of elasticity [e.g. Young's ( $Y$ ),  $G$  or  $E_H$  and  $E_B$  moduli], respectively [43-54, 57-60].

Other additional parameter, i.e. a piezomagnetic dynamics, may be measured as the differences between maximum and minimum values of the electrical impedance on the frequency characteristics, i.e. as the dynamical impedances

$$\hat{Z}_d = \hat{Z}_{\max} - \hat{Z}_{\min}, \quad (6.4)$$

or as diameters of the motional impedance circles [43–45, 59, 60].

In ultrasonic measurements the real and imaginary parts correspond to the sound velocity ( $c$ ,  $c_H$  or  $c_B$ ) and attenuation.

These parameters in non-magnetic materials are known as material constants, and, in classical investigations they are coefficients and are measured as a function of temperature, frequency, stress amplitude, direction, pressure, time (after-effect), treatment history or chemical composition, texture, etc.

### 7. Electrical equivalent circuits and measurement methods

The classic lumped element equivalent circuits of piezomagnetic transducers were discussed by many authors, e.g. [21, 43–45, 48, 50–54, 57–84].

Author of this paper uses series (or parallel) magnetic circuit connected with the parallel mechanical circuit (Fig. 6), e.g. [44, 45, 48, 50, 51, 53, 59, 60, 69, 77].

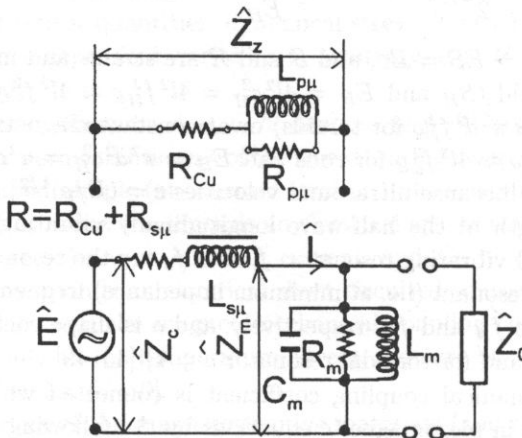


Fig. 6. Equivalent circuits of the piezomagnetic transducers [series-parallel (downer) and parallel-parallel (upper part) with losses.  $\hat{E}$  is the SEM (voltage), applied to transducer and  $\hat{I}$  is the electric current.  $\hat{Z}_c$  is the transducer impedance and  $\hat{Z}_z$  is the electrical (magnetic) impedance, representing the series (lower) circuit,  $R_{Cu} + R_{s\mu} + j\omega L_{s\mu} = R + j\omega L_{s\mu}$ ] or parallel form of the equivalent circuit,  $R_{Cu} + (\omega^2 L_{p\mu}^2 R_{p\mu} + j\omega L_{p\mu} R_{p\mu}^2) / (R_{p\mu}^2 + \omega^2 L_{p\mu}^2)$  of the electrical coil with magnetic core, where  $R_{Cu}$  are electrical losses in the winding and  $R_{s\mu}$  and  $R_{p\mu}$  are magnetic losses in the core for the series and parallel circuits, respectively, and e.g. the resistance  $R = R_{Cu} + R_{s\mu}$ . The reactance component, connected with magnetic permeability of the magnetic circuit with core, is represented by  $j\omega L_{s\mu}$  for series electrical circuit or by  $j\omega L_{p\mu}$  for parallel electrical circuit.  $\hat{Z}_m$  is the motional impedance, i.e. mechanical impedance of the transducer transformed to the electrical circuit,  $R_m$ ,  $C_m$  and  $L_m$  represent mechanical losses ( $Q$ ) or internal friction ( $Q^{-1}$ ), mass and elasticity of the transducer, respectively.  $\hat{Z}_0$  is impedance of the loading [43–45, 50–54, 59, 60, 77].

$\hat{Z}_m$  in Fig. 6 is the mechanical impedance transformed to the electrical circuit.  $R_m$  represents internal friction losses ( $Q^{-1}$ ) and at the resonance is equal to dynamical impedance  $\hat{Z}_d$  and diameter ( $D$ ) of the motional impedance circle, i.e.  $R_m = \hat{Z}_d = D$  (Fig. 7).  $C_m$  and  $L_m$  are mass and elasticity of transducer, respectively.  $\hat{Z}_c$  is the resonator (transducer) impedance and  $\hat{Z}_z$  is the electrical (magnetic) impedance representing the series [(s) or parallel (p)] form of the equivalent circuit of electrical losses  $R = R_{Cu} + R_\mu$  (where  $R_{Cu}$  are the losses in the winding and  $R_\mu$  (s or p) the losses in magnetic core) and reactance component  $j\omega L_\mu$  (where  $L_\mu$  (s or p) is inductance of the coil with the core).  $\hat{Z}_0$  is an loading impedance.

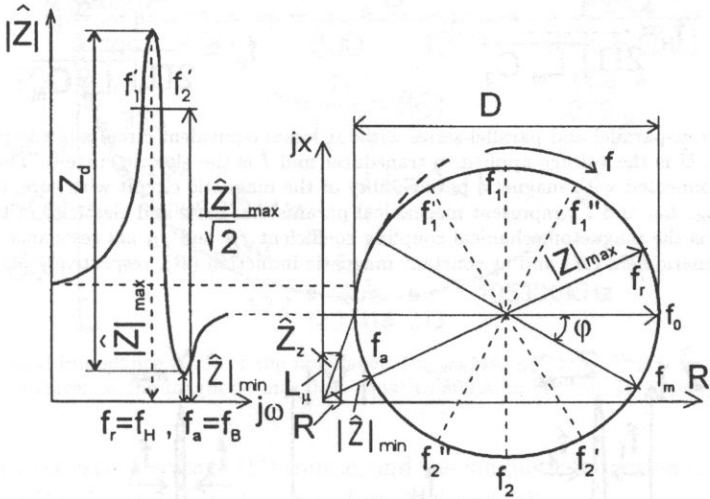


Fig. 7. Resonant-antiresonant impedance characteristic  $Z = |\hat{Z}|$  and motional impedance circle diagram plotted vs. frequency  $f$ .  $\hat{Z}_{\max}$  and  $\hat{Z}_{\min}$  are the maximum and minimum values of impedance for resonance ( $f_r \approx f_H$ ) and antiresonance frequencies ( $f_a \approx f_B$ ).  $f_0$  and  $f_m$  are mechanical resonance frequencies (without or with magnetic losses), and  $f_1, f_2, f_1', f_2', f_1'', f_2''$  are the quadrantal frequencies, respectively.  $f_H$  and  $f_B$  are the resonance frequencies at constant magnetic field ( $H$ ) and at constant magnetic induction ( $B$ ), respectively. For ideal (without losses) transducers  $f_H = f_r$  and  $f_B = f_a$ . At resonance  $\hat{Z}_{\max} - \hat{Z}_{\min} = \hat{Z}_d = R_m$ , where  $Z_d$  is the dynamical impedance equivalent to the piezomagnetic dynamics. For series magnetic circuit  $\hat{Z}_z = R_{Cu} + R_{s\mu} + j\omega L_{s\mu} = R + j\omega L_{s\mu}$  and for parallel form of the equivalent circuit,  $\hat{Z}_z = R_{Cu} + (\omega^2 L_{p\mu}^2 R_{p\mu} + j\omega L_{p\mu} R_{p\mu}^2) / (R_{p\mu}^2 + \omega^2 L_{p\mu}^2)$  [43-45, 50-54, 59, 60, 77].

$\hat{Z}_{\max} - \hat{Z}_{\min} = \hat{Z}_d$  are maximum and minimum values of the impedance for resonance ( $f_r$ ) and antiresonance ( $f_a$ ) frequencies and dynamical impedance,  $f_0$  and  $f_m$  are mechanical resonance frequencies (without or with magnetic losses),  $f_1, f_2, f_1', f_1'', f_2'$  and  $f_2''$  are quadrantal frequencies (Fig. 7).  $Q_H$  and  $Q_B$  are magnetomechanical quality factors for characteristics of the impedance or admittance (Fig. 9) and  $k$  is the magnetomechanical coupling coefficient (Figs. 8-10). Some mathematical dependencies are given in Figs. 8 and 9.

The parallel resonant circuit represents the compliance and the total mass of the resonator (Figs. 6 and 8). The frequency of the free mechanical vibration of the open-circuited transducer, i.e. when internal impedance is going to infinity ( $Z \rightarrow \infty$ ) is the

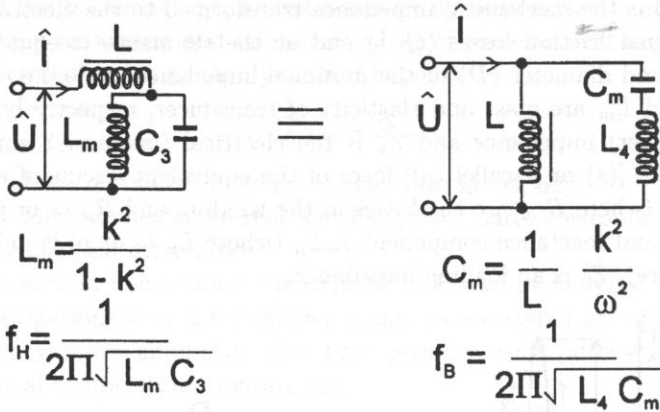


Fig. 8. The series-parallel and parallel-series without losses equivalent circuits of the piezomagnetic transducers.  $\hat{U}$  is the voltage applied to transducer and  $\hat{I}$  is the electric current. The reactance component, connected with magnetic permeability of the magnetic circuit with core, is represented by  $j\omega L$ .  $C_3$ ,  $C_m$ ,  $L_m$  and  $L_4$  represent mechanical parameters: mass and elasticity of the transducer, respectively.  $k$  is the magnetomechanical coupling coefficient  $f_H$  and  $f_B$  are resonance frequencies at constant magnetic field ( $H$ ) and at constant magnetic induction ( $B$ ), respectively [43, 59, 60, 77].

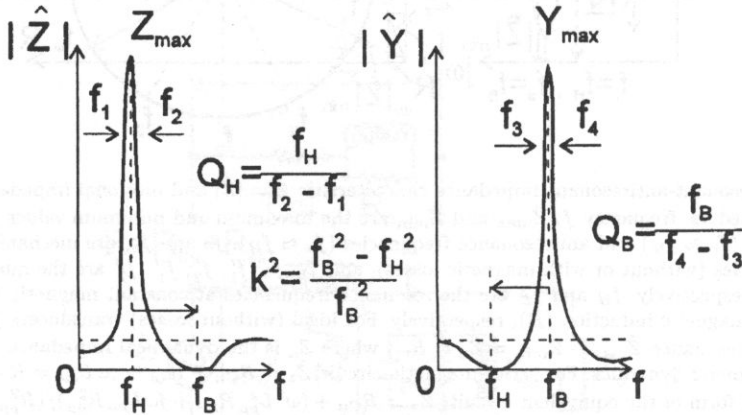


Fig. 9. The impedance  $Z = |\hat{Z}|$  and admittance  $Y = |\hat{Y}|$  frequency characteristics for nearly ideal circuits.  $Z_{\max}$  and  $Y_{\max}$  are the maximum values of the impedance and admittance for resonance ( $f_r \approx f_H$ ) and antiresonance frequencies ( $f_a \approx f_B$ ). The magnetomechanical coupling coefficient  $k$ ,  $Q_H$  and  $Q_B$  are calculated from the resonance  $f_H$  (at constant magnetic field  $H$ ) and antiresonance  $f_B$  (at constant magnetic induction  $B$ ) frequencies and from the quadrantal frequencies:  $f_1, f_2, f_3, f_4$ , respectively [43, 59, 60, 77].

parallel resonance frequency ( $f_r \approx f_H$ , as in Figs. 7, 9 and 10). This is the case of the constant current ( $I$ ) source, and the magnetic field ( $H = IN/l_m$ , where  $N$  is turn number and  $l_m$  is the length of the magnetic path of the coil), is proportional to this current.

The series resonance (or antiresonance) frequency is equal to the frequency of the short-circuited vibrator, i.e. when internal impedance is going to the zero ( $Z \rightarrow 0$ ). This



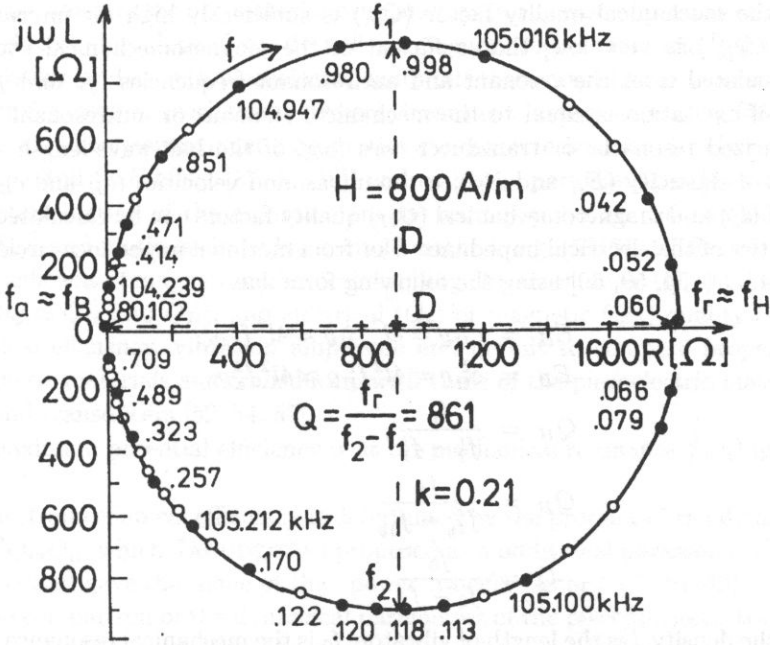


Fig. 10. Motional impedance circle of the nonloaded  $\text{Ni}_{0.965}\text{Mn}_{0.02}\text{Co}_{0.015}\text{Fe}_2\text{O}_4$  ferrite toroidal transducer at the magnetic bias field equal to 800 A/m [44, 48, 53, 59, 60].

is the case of the constant voltage ( $U$ ) source, and the magnetic induction is proportional to the voltage ( $B \sim U$ , and  $f_a \approx f_B$ , as in Figs. 7, 9 and 10).

The series-parallel and parallel-series equivalent circuits of the piezomagnetic transducers without losses are given in Fig. 8 [43, 44, 48, 59, 60].

The connections between the resonant-antiresonant the impedance and admittance characteristics, plotted as the functions of frequency are presented in Fig. 9, e.g. [43-45, 59, 60].

The motional impedance circle diagram (Fig. 7) is more universal and provides more information [43-45, 59, 60, 77]. It is possible to obtain from the motional impedance circle not only the frequencies of the magnetomechanical resonances for maximum and minimum impedance, i.e.  $f_r$  and  $f_a$ , as in classical resonance-antiresonance method (Figs. 7 and 9), but also the frequencies of the mechanical resonance for maximum resistance  $f_0$  for the circuit without magnetic losses (Figs. 7 and 10), or  $f_m$  for the circuit with magnetic losses (Fig. 7) [43-45, 50-54, 59, 60].

The magnetomechanical quality factor  $Q [= f_r / (f_2' - f_1')$  or  $f_r / (f_2 - f_1)$  for  $Z_{\max} / 2^{1/2}$ ] or magnetomechanical internal friction coefficient  $Q^{-1}$  may be obtained from both methods (Figs. 3, 5-7) but the accuracy of the resonant-antiresonant method is very low (about 20%). The accuracy of the frequency measurement in motional impedance circle method is very high (about  $10^{-5} - 10^{-6}$ ) [48, 59, 60, 77]. From the last method it is possible to obtain also the mechanical quality factor  $Q_m$  and the mechanical internal friction coefficient ( $Q_m^{-1}$ ).

When the mechanical quality factor ( $Q_m$ ) is sufficiently high [or internal friction coefficient ( $Q_m^{-1}$ ) is very low] the coefficient of the magnetomechanical coupling ( $k$ ) can be calculated from the resonant and antiresonant frequencies ( $f_r$  and  $f_a$ ). If the frequency of excitation is equal to the mechanical resonant or antiresonant frequency of the polarized resonator or transducer core (e.g. of the half-wave-length resonator) the moduli of elasticity ( $E_H$  and  $E_B$ ) or the ultrasound velocities ( $c_H$  and  $c_B$ ) and the mechanical ( $Q$ ) and magnetomechanical ( $Q_H$ ) quality factors can be calculated from the characteristics of the electrical impedance  $Z$  or from motional impedance circles (Figs. 7, 9, 10) [43–45, 50–54, 59, 60] using the following formulae:

$$E_H = c_H^2 \varrho = 4l^2 f_H^2 \varrho \approx 4l^2 f_r^2 \varrho, \quad (7.1)$$

$$E_B = c_B^2 \varrho = 4l^2 f_B^2 \varrho \approx 4l^2 f_a^2 \varrho, \quad (7.2)$$

$$Q_H = \frac{f_r}{f'_2 - f'_1}, \quad (7.3)$$

$$Q_B = \frac{f_a}{f'_{4y} - f'_{3y}}, \quad (7.4)$$

$$Q = \frac{f_0}{f_2 - f_1}, \quad (7.5)$$

where  $\varrho$  is the density,  $l$  is the length of vibrator,  $f_0$  is the mechanical resonance frequency (for the maximum resistance) and  $f_1, f_2, f'_1, f'_2, f'_3$  and  $f'_4$  are the quadrant frequencies (Figs. 7, 9 and 10), respectively.

In the case of very low electrical and magnetic losses the diameter of the motional impedance circle will be on the  $R$  axis or very closely parallel to it (e.g. in ferrites [43, 44, 53, 59, 60, 77] or some metallic glasses [59, 60]) and then  $f_H \approx f_r \approx f_0$  or  $f_B \approx f_a$  (Fig. 10).

In the case of high electrical and magnetic losses, in this discussion must be taken into account a loss angle  $\varphi$  and then the frequency of the mechanical resonance  $f_m$  will be over  $f_r$  (for maximum impedance) and  $f_0$  (for maximum resistance, Fig. 7).

The amplitude of the mechanical vibrations at the resonant frequencies will be multiplied by the mechanical quality factor ( $Q_m$ ).

Magnetostrictive materials for good piezomagnetic transducers should exhibit magnetostriction higher than  $15 \times 10^{-6}$ , magnetic saturation higher than 1 T, the Curie temperature higher than 200–300°C, electrical resistivity higher than  $1 \mu\Omega\text{m}$  and stable temperature coefficients of the static and dynamical properties, e.g. [44, 50–54].

The efficiency

$$\eta \approx 1 - 2(k^2 Q_m Q_\mu)^{-1/2} \quad (7.6)$$

may be determined from the resonant-antiresonant curves or from the motional circle diagrams using the equivalent circuits for piezomagnetic resonators [44, 52–54, 83].

To fundamental dynamical parameters of the piezomagnetic materials and transducers, except the magnetomechanical coupling coefficient ( $k$ ), mechanical and magnetic quality factors ( $Q_m$  and  $Q_\mu$ ), efficiency ( $\eta$ ), belong also an amplitude of mechanical vibrations ( $A$ ) and other coefficient occurring in piezomagnetic equations (e.g.  $E_H, E_B, \mu_T, \mu_S, d, e, g, h$ ).

These parameters are not material constants, as in the case of piezoelectric materials and transducers, but depend on dc and ac magnetic field, magnetic, mechanical and thermal history, and as with piezoelectric materials these may change as a function of the temperature, pressure, mechanical stresses, treatment history and so on [43–45, 50–54, 59, 60, 85, 86]. The piezomagnetic properties are characterized also by characteristics of elasticity moduli ( $\Delta E$  effect [17, 31–40]) and magnetic permeabilities [43–54, 86].

For determination of these parameters the most popular are the resonant-antiresonant method and motional impedance circle method, e.g. [59–61, 67–83, 86].

The coefficient of magnetomechanical coupling ( $k$ ), mechanical ( $Q_m$ ), or magnetomechanical ( $Q \approx Q_H$  and  $Q_B$ ), and electrical ( $Q_e$ ) or magnetic ( $Q_\mu$ ) quality factors, electroacoustical efficiency, vibration amplitude etc. permit to compare properties of the piezomagnetic materials and transducers with these of the piezoelectric materials, components and transducers [52–54, 57].

The maximum potential efficiency is at the mechanical resonance  $f_m$  (Fig. 7) [43, 59, 60, 83].

The electromechanical efficiency is determined by the product of the dynamical coefficients  $k^2 Q_m Q_\mu$ , which JAGODZIŃSKI proposed as a additional parameter of ultrasound transducers and gave the name of the "power transfer factor ( $n$ )" [81–83].

For the comparison of the dynamical parameters of the piezomagnetic transducers or materials it should be useful also the motional impedance or the frequency characteristics of the electrical impedance.

The piezomagnetic dynamics, i.e. the difference between maximum and minimum impedances,  $Z_d = Z_{\max} - Z_{\min}$ , or a diameter of the motional impedance circle ( $D$ ) (Figs. 7, 9 and 10) is also the important parameter [59, 60].

The admittance characteristics and motional circuit diagrams of the piezoelectric materials and transducers are characteristic for the given materials or transducers at chosen temperature or mechanical conditions but for the piezomagnetic materials the similar characteristics of the impedance are changing with the magnetic, mechanical or thermal conditions [43–46, 48, 50–54, 59, 60, 85].

For the given materials or transducers, e.g. at the room temperature, there are families of motional impedance circles which depend on magnetic fields, load, stresses and so on., e.g. [44, 59, 60].

In the great part of the papers there are given only as the examples the motional circles without comparisons as the functions of magnetic fields, temperatures and etc.

## 8. Final remarks and conclusions

The piezomagnetic parameters, such as the coefficient of magnetomechanical coupling mechanical and electrical or magnetic quality factors, electroacoustical efficiency, vibration amplitude, permit to compare properties of the piezomagnetic materials and transducers with these of the piezoelectric materials and transducers.

For a comparison of the dynamical properties of the piezomagnetic materials and transducers the piezomagnetic dynamics may be the additional parameter useful in the-

ory and applications. The maximum values of the dynamical impedance are at the same or something different magnetic bias fields as in the case of the maximum magnetomechanical coupling or the minima of the elasticity moduli or ultrasound velocities.

Very important in these investigations are their mechanical, thermal and magnetic histories. The influence of heat treatment, especially above the Curie temperature (but below the crystallization temperature in metallic glasses) is very useful [40, 41, 51, 85].

Also annealing in perpendicular or longitudinal magnetic fields or stress-annealing improve the piezomagnetic properties of the magnetostrictive materials.

### References

- [1] G. GILBERT, *De magnete, magneticisque corporibus, et de magno magnete tellure*, *Physiologia nova*, Petrus Short, Londini 1600.
- [2] C.G. PAGE, *Amer. J. Sci.*, **32**, 396–397 (1837).
- [3] C.G. PAGE, *Ann. Phys. Chemie*, **43**, 411 (1838).
- [4] C.G. PAGE, *Ann. Phys. Chemie*, **63**, 530 (1844).
- [5] S.R. WILLIAMS, *Some experimental methods in magnetostriction*, **14**, 5, 383–408 (1927)
- [6] J.P. JOULE, *On a new class of magnetic forces*, *Ann. Electr. Magn. Chem.*, **8**, 219–224 (1842).
- [7] J.P. JOULE, *On the effects of magnetism upon the dimensions of iron and steel bars*, *Phil. Mag.*, [3], **30**, 2, 76–87; 4, 225–241 (1847).
- [8] G. WERTHEIM, *Recherches sur l'élasticité, 3. De l'influence du courant galvanique et de l'électromagnétisme sur l'élasticité des métaux*, *Ann. Chim. Phys.*, [3], **12**, 610–624 (1844).
- [9] A. GUILLEMIN, *Observations relatives au changement qui se produit dans l'élasticité d'un barreau de fer doux sous l'influence de l'électricité*, *Compt. Rend. Ac. Sci. Paris*, **22**, 264–265, 432–433 (1846).
- [10] C. MATTEUCCI, *Mémoire sur le magnétisme développé par le courant électrique*, *Compt. Rend. Acad. Sci. Paris*, **24**, 301–302 (1847).
- [11] E. VILLARRI, *Intorno alle modificazioni del momento magnetico di una verga di ferro e di acciaio, prodotte per la trazione della modesina e pel passaggio di una corrente elettrica attraverso la stessa*, *Nuovo Cimento*, **20**, 317–362 (1865).
- [12] G.H. WIEDEMANN, *Magnetische Untersuchungen*, *Ann. Phys. Chem.*, **117**, 193–217 (1862).
- [13] N. NAGAOKA, K. HONDA, *On magnetostriction*, *Phil. Mag.*, [5], **46**, 280, 261–290 (1898).
- [14] W.F. BARRETT, *On the alternations in the dimensions of the magnetic metals by the act of magnetisation*, *Nature*, **26**, 585–586 (1882).
- [15] W.L. ROZING, *Zh. R. CH. O. (Fiz.)*, **26**, 253 (1894).
- [16] C.E. GUILLAUME, *Action des additions métallurgiques sur l'anomalie de dilatabilité des aciers au nickel*, *Compt. Rend. Ac. Sci. Paris*, **170**, 1433 (1897).
- [17] K. HONDA, S. SHIMIZU, S. KUSAKABE, *Change of the modulus of rigidity of ferromagnetic substances by magnetization*, *Phil. Mag.*, [6], **4**, 23, 537–546 (1902).
- [18] W. VOIGT, *Ueber Pyro- und Piezomagnetismus der Krystalle*, *Ann. Physik*, [4], **9**, 9, 94–114 (1902).
- [19] W. VOIGT, *Lehrbuch der Krystalphysik*, B.G. Teubner, Leipzig 1910.
- [20] K. HONDA, S. SHIMIZU, *Note on the vibration of ferromagnetic wires placed in a varying magnetic field*, *Phil. Mag.*, [6], **4**, 24, 645–652 (1902).

- [21] G.W. PIERCE, *Magnetostriction oscillators. An application of magnetostriction to the control of frequency of audio and radio electric oscillations, to the production of sound, and to the measurement of the elastic constants of metals*, Proc. Amer. Acad. Arts Sci., **63**, 1, 1-47 (1928).
- [22] B.H. BARKHAUSEN, *Zwei mit Hilfe der neuen Verstärker entdeckte Erscheinungen*, Phys. Z., **20**, 17, 401-403 (1919).
- [23] C.E. GUILLAUME, *Discovery of the anomaly of the nickel steels*, Proc. Phys. Soc., **32**, 374-404 (1920).
- [24] W.L. WEBSTER, *Magnetostriction and change of resistance in single crystals of iron and nickel*, Proc. Phys. Soc., **42**, 5 (235), 431-440 (1930).
- [25] H.E. BRUKE, *Handbook of Magnetic Phenomena*, Van Nostrand Reinhold Co., New York 1986.
- [26] I.E. DZIALOSHINSKII, *On the question of piezomagnetism* [in Russian], Zh. Exper. Theor. Phys., **33**, 3 (9) 807-808 (1958).
- [27] A.S. BOROVNIK-ROMANOV, *Piezomagnetism in antiferromagnetic cobalt and manganese fluorides*, Sov. Phys. JETP, **36** (9), 6, 1390-1391 (1959).
- [28] P.W. ANDERSON, R.R. BIRSS and R.A.M. SCOTT, *Linear magnetostriction in hematite*, Proc. Int. Conf. on Magnetism, Nottingham 1964, pp. 587-590.
- [29] I.J. GARSHELIS, *A versatile magnetostrictive displacement transducer*, [in:] AIP Conf. Proc., **29**, J.J. BECKER, G.H. LANDER and J.J. RHYNE [Eds.], 1975, pp. 639-644.
- [30] A.E. LORD, *Acoustic emission*, [in:] Physical Acoustics, vol. 11, W.P. MASON and N.R. THURSTON [Eds.], Academic Press, New York 1975, pp. 290-329.
- [31] N.S. AKULOV, *Ferromagnetizm*, GITTL, Moskwa 1939.
- [32] R. BECKER, W. DÖRING, *Ferromagnetismus*, Springer Verlag, Berlin 1939.
- [33] L.F. BATES, *Modern Magnetism*, Cambridge Univ. Press, Cambridge 1948.
- [34] R.M. BOZORTH, *Ferromagnetism*, D. van Nostrand Co., Inc., Princeton 1951.
- [35] E.W. LEE, *Magnetostriction and magnetomechanical effect*, Rep. Prog. Phys., **18**, 184-229 (1955).
- [36] K.P. BIELOW, *Phenomena in Magnetic Materials* [in Polish], PWN, Warszawa 1962.
- [37] E. KNELLER, *Ferromagnetismus*, Springer, Berlin 1962.
- [38] S.V. VONSOVSKII, *Magnetizm*, Izd. Nauka, Moskwa 1971.
- [39] C. HECK, *Magnetic Materials and their Applications*, Butterworths, London 1974.
- [40] E. DU TRÉMOLET DE LACHEISSERIE, *Magnetostriction: Theory and Applications of Magnetoelasticity*, CRC Press Inc., Boca Raton 1993, pp. 1-409.
- [41] R.C. O'HANDLEY, *Magnetostriction of metallic glasses*, [in:] Amorphous Magnetism II, R.A. LEVY and R.A. HASEGAWA [Eds.], Plenum, New York 1977, pp. 379-392.
- [42] Z. KACZKOWSKI, *Linear magnetostriction of the amorphous magnetics* [in Polish], Biuletyn Informacyjny Elektroniczne Podzespoły Bierne, 1-3, 13-36 (1984).
- [43] C.M. VAN DER BURGT, *Dynamical physical parameters of the magnetostrictive excitation of extensional and torsional vibration in ferrites*, Philips Res. Rep., **8**, 2, 91-132 (1953).
- [44] Z. KACZKOWSKI [Ed.], *Piezomagnetic Materials and their Applications* [in Polish], PWN, Warszawa 1978, pp. 1-813.
- [45] Z. KACZKOWSKI, *Applications of the magnetostrictive and piezomagnetic phenomena in radio-electronics and metrology*, [in:] VI. Vedecka Konferencia Elektrotechnickej Fakulty, Košice, 1992, Zborník Prednášok, Fyzika, Technická Univerzita v Košicach, Košice 1992, pp. 132-155.
- [46] Z. KACZKOWSKI, *Piezomagnetic equations and their coefficients* [in Polish], Rozpr. Elektrotech., **7**, 2, 245-275 (1962).

- [47] Z. KACZKOWSKI, *The coefficients of magnetostrictive equations and their relationship*, Proc. Vibr. Probl., **2**, 4, 457–468 (1961).
- [48] Z. KACZKOWSKI, *Piezomagnetic coefficients of the magnetostrictive ferrites and their magnetic hysteresis* [in Polish], Prace Instytutu Fizyki PAN, **39**, 1–378 (1972).
- [49] Z. KACZKOWSKI, *Magnetomechanical coupling in transducers*, Arch. Acoust., **6**, 4, 385–400 (1981).
- [50] Z. KACZKOWSKI, *Magnetomechanical interactions*, Mater. Sci. Forum, 119–121, 591–602 (1993).
- [51] Z. KACZKOWSKI, *Magnetomechanical properties of rapidly quenched materials*, Mater. Sci. Eng. A, 226–228, 614–625 (1997).
- [52] Y. KIKUCHI [Ed.], *Ultrasonic Transducers*, Corona Publ. Co. Ltd., Tokyo 1969.
- [53] W. PAJEWSKI, Z. KACZKOWSKI, E. STOLARSKI, *Piezotronische Bauelemente*, [in:] Handbuch der Elektronik, Franzis-Verlag, München 1979, pp. 269–337.
- [54] D.A. BERLINCOURT, D.R. CURRAN, H. JAFFE, *Piezoelectric and piezomagnetic materials and their function in transducers*, [in:] Physical Acoustics, W.P. MASON [Ed.], Academic Press, New York 1964, vol. 1A, pp. 169–270.
- [55] G. ŁYPACEWICZ, L. FILIPCZYŃSKI, *Measurements of the clamped capacitance  $C_0$  and the electromechanical coupling coefficient  $k_t$  of piezoelectric ceramic transducers under mechanical load*, Acustica, **25**, 1, 64–68 (1971).
- [56] L. FILIPCZYŃSKI, *Mechanism of occurrence of vibrations in magnetostrictive transducers*, Proc. Vibr. Probl., **1**, 1, 15–28 (1960).
- [57] H.W. KATZ [Ed.], *Solid State Magnetic and Dielectric Devices*, J. Wiley & Sons Inc. Publ., New York 1959.
- [58] Z. KACZKOWSKI, *Stress sensitivity in Ni-Mn, Ni-Mn-Co and Ni-Mn-Co-Cu ferrites*, J. Magn. Magn. Mater., **41**, 1–3, 338–340 (1984).
- [59] Z. KACZKOWSKI, *Piezomagnetic dynamics*, [in:] XLIII Open Seminar on Acoustics, OSA'96, Ustroń, M. ROCZNIK (Ed.), Polskie Towarzystwo Akustyczne – Oddział Górnośląski, Gliwice 1996, 1, pp. 317–324.
- [60] Z. KACZKOWSKI, *Piezomagnetic dynamics as a new parameter of magnetostrictive materials and transducers*, Bull. Pol. Ac. Sci., Tech. Sci., **45**, 1, 19–42 (1997).
- [61] S. BUTTERWORTH, F.D. SMITH, *The equivalent circuit of the magnetostriction oscillator*, Proc. Phys. Soc., **43**, 2, 166–185 (1931).
- [62] W.P. MASON, *Physical Acoustics and the Properties of Solids*, D. van Nostrand, New York 1958.
- [63] E.G. RICHARDSON, *Technical Aspects of Sound*, Elsevier Publ. Co., Amsterdam 1957.
- [64] J.W. MILES, *Applications and limitations of mechanical-electrical analogies*, J. Acoust. Soc. Am., **14**, 2, 183–192 (1943).
- [65] H.F. OLSON, *Dynamical Analogies*, Van Nostrand, New York 1953.
- [66] I. MAŁECKI, *Physical Foundation of Technical Acoustics*, Pergamon Press, Oxford 1969.
- [67] H. NØDTVEDT, *Analysis of magnetostrictive transducers*, Acustica, **4**, 4, 432–438 (1954).
- [68] J.P. PERKINS, *Analysis of piezomagnetic vibrators*, Ultrasonics, **2**, Oct.-Dec., 193–198 (1964).
- [69] Z. KACZKOWSKI, *A simplified measuring set for determination of mechanical resonance frequency and magnetomechanical coupling coefficient  $k$  of magnetostrictive materials* [in Polish], Archiwum Elektrot., **11**, 3, 635–636 (1962).
- [70] M.J. IDE, *Measurements on magnetostriction vibration*, Proc. Inst. Radio Eng., **19**, 7, 1216–1232 (1931).
- [71] F.P. FINLON, *Motional impedance measurements on a magnetostrictive system*, J. Acoust. Soc. Am., **21**, 3, 177–182 (1949).

- [72] O.K. MAWARDI, *Measurement of acoustic impedance*, J. Acoust. Soc. Am., **21**, 2, 84–91 (1949)
- [73] H.J. ROUND, *Magnetostrictive transducer measurements*, Wireless Engr., **29**, 343, 101–105 (1952).
- [74] W. DIETRICH, *Ein Ortskurvenschreiber für Tonfrequenzen*, Funk u. Ton, **8**, 404–413 (1953).
- [75] M.T. PIGOTT, P.M. KENDING, *Rapid method of evaluating magnetostrictive materials for electromechanical transducers*, J. Acoust. Soc. Am., **26**, 6, 974–976 (1954).
- [76] C.A. CLARK, *The dynamic magnetostriction of nickel-cobalt alloys*, Br. J. Appl. Phys., **7**, 10, 355–360 (1956).
- [77] Z. KACZKOWSKI, *Ultrasonic investigation methods of the piezomagnetic materials*, [in:] II Bilateral Polish-German Symposium on Ultrasonic Measurement Technics in Science & Practice, Gliwice-Wisła, 1994, Upper Silesian Division of the Polish Acoustical Society, Gliwice 1994, pp. 29–40.
- [78] Z. KACZKOWSKI, *A method and measurement set for temperature investigation of internal friction and  $\Delta E$ -effect of magnetostrictive cylindrical samples in the range from 10 to 40 kHz* [in Polish], [in:] Tarcie wewnętrzne i opóźnienie magnetyczne w ciałach stałych, J.W. MOROŃ and J. ILCZUK, [Eds.], Prace Nauk. Uniw. Śląskiego w Katowicach, 738, Katowice 1985, pp. 117–125.
- [79] S. MANDACHE, *Coefficient of the magneto-mechanical coupling of magnetostrictive ferrite*, Elektrotehnica, **19**, 11, 412–415 (1971).
- [80] K.B. HATHAWAY, M.L. SPANO, *Measurement of high magnetomechanical coupling factors by resonance techniques*, J. Appl. Phys., **55**, 6, 1765–1767 (1984).
- [81] Z. JAGODZIŃSKI, *On the adequacy of equivalent circuits for piezomagnetic transducers*, Acustica, **21**, 5, 283–287 (1969).
- [82] Z. JAGODZIŃSKI, *Mechanical or electromechanical resonance*, Acustica, **23**, 6, 353–361 (1970).
- [83] Z. JAGODZIŃSKI, *Efficiency of the piezomagnetic transducers* [in Polish], [in:] Materiały piezomagnetyczne i ich zastosowania, Z. KACZKOWSKI [Ed.], PWN, Warszawa 1978, pp. 586–594.
- [84] Z. KACZKOWSKI, *Equivalent circuits of the resonators and piezomagnetic resonant transducers and their circuits* [in Polish], [in:] Materiały piezomagnetyczne i ich zastosowania, Z. KACZKOWSKI [Ed.], PWN, Warszawa 1978, pp. 578–585.
- [85] Z. KACZKOWSKI, *Effect of the heat treatment on piezomagnetic properties*, Int. J. Appl. Electromagn. in Mater., **5**, 2, 229–240 (1994).
- [86] H.T. SAVAGE, M.L. SPANO, *Theory and application of highly magnetoelastic Metglas 2605-SC*, J. Appl. Phys., **53**, 11, 2, 8092–8097 (1982).