APPEARANCE OF THE ACOUSTIC AND ELECTROMAGNETIC LATERAL WAVE IN PROPAGATION PHENOMENA

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The paper presents the appearance of lateral waves of the acoustic and electromagnetic type in various physical phenomena, from the earthquakes up to ultra-high frequency radio-wave propagation in stratified medium. The lateral wave appears in the process of wave refraction to the medium characterized by higher wave speed, if the angle of incidence exceeds the critical value. The lateral wave arrives at the receiver first and in some cases is the only remaining wave at long distances, thus it plays an important role in early warning, wireless communication in lossy environments or ground penetrating radar applications, when geometric optic model leads to significant errors.

Key words: acoustic, electromagnetic lateral wave, two media interface, multilayered media.

1. Introduction

The intent of the paper is to provide a better understanding of the physical background of the lateral wave appearance and to stress its meaning in appropriate interpretation of propagation phenomena. Negligence of the lateral wave in applied theoretical model may lead to significant discrepancy between predictions of the theory and the experimental data, what will be presented on examples for both the acoustic and electromagnetic lateral wave. Thus a necessity had arisen to develop adequate diffraction model of wave propagation, accounting for real features of the environment in which the propagation takes place.

According to Snell's law, if the angle of incidence is less than the critical angle, a reflected wave arises together with a transmitted wave. For waves incident at an angle greater than the critical one, a phenomenon of total internal reflection is observed.

Consider a plane wave falling on the interface separating two media at the angle exactly equal to the critical angle. According to the Fermat principle on extreme propagation time, together with the Huygens principle, for the angle of incidence equal to the critical angle the incoming wave can propagate over some distance in the lower medium with greater velocity and come back to the upper medium. In fact such wave is observed and called the lateral wave.

Considering the propagation of a spherical wave in layered media, application of the model requires taking into account all the phenomena described above, with reflection coefficient depending on the angle of incidence. The problem was formulated and solved by BREKHOVSKIKH [1–2] by means of the fundamental papers of SOMMERFELD [3], who had developed the far-field solution for dipole source and WEIL [4], who presented a method of expansion of a spherical wave into plane waves.

Today the lateral wave of acoustic or electromagnetic type is a subject of widespread investigations in the fields of seismology, radio-communication, underground objects detection and other shallow-water propagation problems, and also medicine. This will be a subject of further considerations.

2. The two-media interface

The reflection of a spherical wave at a plane boundary between two media [1–2, 5–6] is considered. The point source (Fig. 1) is located at the height z_s in a medium characterised by the density ρ_1 and the speed of wave c_1 , equal to ρ_2 and c_2 in the lower medium, respectively. We assume that $c_2 > c_1$.



Fig. 1. Geometry of the reflected field. The boundary between two media is located at z = 0. The lower medium is characterised by greater velocity, thus total reflection at the media interface takes place.

The plane-wave reflection coefficient is [1]

$$V(\theta) = \frac{m\cos\theta - \sqrt{n^2 - \sin^2\theta}}{m\cos\theta + \sqrt{n^2 - \sin^2\theta}},\tag{1}$$

where $m = \rho_2/\rho_1$ for acoustic waves, while for electromagnetic waves $m = n^2$ for vertical and m = 1 for horizontal polarisation (Fresnel reflection coefficients), $n = c_1/c_2$ for both types of waves. Remember that usually the refraction coefficient for electromagnetic waves is related to vacuum/air and then, assuming the second medium to be air, we have $n = 1/n_0$ where $n_0 = c_0/c_1$ is the refraction coefficient of light. For n < 1 the critical angle is defined as $\theta_{cr} = \arcsin(n)$. For vertical polarisation and the angle of incidence equal to the Brewster angle $(n = \tan \theta_B)$, the reflection coefficient equals zero.

3. Plane-wave representation of a spherical wave

The first step is to present the plane wave integral representation of a spherical wave

$$\Phi_0(R) = \frac{e^{ikR}}{R} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{1}{2\pi k_z} e^{i(k_x x + k_y y + k_z z)} dk_x dk_y, \qquad z \ge 0,$$
(2)

where $k = \omega/c$ is the wave number, ω the wave frequency and c the speed of wave [1, 6–7].

The spherical wave is decomposed into an infinite number of plane waves incident at the interface at angle θ and expressed as a contour integral of complex variable θ :

$$\Phi_0(R) = ik \int_{0}^{\pi/2 - i\infty} J_0(kr\sin\theta) e^{ikz\cos(\theta)}\sin\theta d\theta,$$
(3)

where $r = R \sin \alpha$, $z = R \cos \alpha$, $J_0(w)$ denotes the Bessel function of order zero.

Each of these contributing plane waves obeys the Snell's law, what allows to determine the reflected field, which is calculated by incorporating the reflection coefficient into the integrand of (3):

$$\Phi_{\rm ref}(R,\alpha) = ik \int_{0}^{\pi/2 - i\infty} J_0(kr\sin\theta) e^{ikz\cos\theta} V(\theta)\sin\theta d\theta.$$
(4)

Equation (4) is the exact formula for the potential of a reflected wave. Expressing the Bessel function by combination of Hankel's functions: $J_0(w) = 1/2 [H_0^{(1)}(w) + H_0^{(2)}(w)]$ and extending, due to the symmetry properties, the integration limits in (4) to $\pi/2 - i\infty, -\pi/2 + i\infty$, the last formula takes the form adequate for future application, since asymptotic form [8] of the Hankel function can be easily incorporated into integrand (compare Eq. (10)).

$$H_0^{(1)}(w) = \sqrt{\frac{2}{\pi w}} e^{i(w-\pi/4)} \left(1 + \frac{1}{8iw} + \dots\right) \cong \sqrt{\frac{2}{\pi w}} e^{i(w-\pi/4)}.$$
 (5)

4. Potential of acoustic and electromagnetic field

Assume a point source of monochromatic wave of a given frequency ω . Acoustic potential at a distance R from the source is

$$\Phi_0(R) = A \frac{e^{i(kR-\omega t)}}{R},\tag{6}$$

where A is the amplitude. The acoustic pressure in a medium of density ρ is defined as

$$p_{ac} = \rho \frac{\partial}{\partial t} \Phi_0. \tag{7}$$

In electrodynamics [9] analogous role play the Hertz vectorial potentials, which for the electric dipole of momentum \mathbf{p} or the magnetic dipole of momentum \mathbf{m} are defined as

$$\mathbf{\Pi} = \mathbf{p} \frac{e^{i(kR-\omega t)}}{R}, \qquad \mathbf{\Pi}_m = \mathbf{m} \frac{e^{i(kR-\omega t)}}{R}.$$
(8)

The Hertz potential Π is connected with scalar Φ and vectorial **A** electromagnetic field potentials [9] by the relations

$$\phi\left(\mathbf{r},t\right) = -\frac{1}{4\pi\varepsilon_{0}} \operatorname{div} \mathbf{\Pi}\left(\mathbf{r},t\right), \qquad \mathbf{A}\left(\mathbf{r},t\right) = \frac{1}{4\pi\varepsilon_{0}c_{0}^{2}} \frac{\partial \mathbf{\Pi}}{\partial t}\left(\mathbf{r},t\right), \tag{9}$$

what ensures fulfilment of the Lorentz gauge condition

$$\operatorname{div} \mathbf{A} + \frac{1}{c_0^2} \frac{\partial \Phi}{\partial t} = 0.$$
 (10)

In nondissipative media (conductivity $\sigma = 0$) the Hertz potential Π , similarly to the scalar Φ and vectorial \mathbf{A} electromagnetic field potentials, fulfil the homogeneous wave equation. The Hertz potential Π_m is connected in a similar way with dual antipotentials and also fulfils the Lorentz gauge condition [9].

The electric and magnetic field vectors are expressed by means of the Hertz vectorial potential of electric dipole Π (all symbols have a common meaning, subscript zero refers to the free field, what means vacuum or air medium)

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \operatorname{rot} \operatorname{rot} \mathbf{\Pi}, \qquad \mathbf{B} = \frac{1}{4\pi\varepsilon_0 c_0^2} \frac{\partial}{\partial t} \operatorname{rot} \mathbf{\Pi}, \tag{11}$$

or the Hertz vectorial potential of the magnetic dipole Π_m

$$\mathbf{D} = -\frac{1}{4\pi\varepsilon_0 c_0^2} \frac{\partial}{\partial t} \operatorname{rot} \mathbf{\Pi}_m, \qquad \mathbf{H} = \frac{1}{4\pi\varepsilon_0} \operatorname{rot} \operatorname{rot} \mathbf{\Pi}_m.$$
(12)

Remember that substituting $\Pi \rightarrow \Pi_m$, simultaneously $\mathbf{E} \rightarrow \mathbf{H}, \ \mathbf{B} \rightarrow -\mathbf{D}$.

In lossy media, which will be of our future interest, the vectorial potential Π obey the so-called inhomogeneous wave equation

$$\Delta \Pi - \frac{1}{c^2} \frac{\partial^2 \Pi}{\partial t^2} = \mu \sigma \frac{\partial \Pi}{\partial t},\tag{13}$$

 σ being the medium conductivity. For time-dependence exp(- $i\omega t$), introducing complex electrical permittivity $\tilde{\varepsilon} = \varepsilon + \varepsilon'$ with imaginary part $\varepsilon' = \sigma/\omega$, the inhomogeneous wave equation reduces to the homogenous one [9].

Thus, it is possible to study the phenomena of wave reflection and refraction without deciding whether the acoustic or electromagnetic field is considered [1, 9].

5. Lateral wave – basic formulae

In this Section the simplest case of the lateral wave appearance is discussed, no matter whether the wave is acoustic or electromagnetic, thus Φ stands for acoustic potential of a point source of unit amplitude (6) or any component of the Hertz vectorial potential of unit momentum (8). The interface between two media is planar, the media are homogenous and nondissipative, thus the propagating wave does not experience any attenuation.

The following expression for the potential has been derived from Eq. (4) by applying the identity $J_0(w) = 1/2[H_0^{(1)}(w) + H_0^{(2)}(w)]$ and substituting under the integral the Hankel's function asymptotic form valid for $|w| \gg 1$ [8]. The spherical wave is then decomposed into an infinite number of plane waves, each of which experience reflection and refraction at the interface between two media [1–2]

$$\Phi_{\rm ref}(R) = c \int_{-\pi/2+i\infty}^{\pi/2-i\infty} e^{ikR\cos(\theta-\alpha)} V(\theta) \sqrt{\sin\theta} d\theta, \quad c = [ik/(2\pi R\sin\alpha)]^{1/2}.$$
 (14)

Regardless of the analytic formula basic for the given assumptions (sound or electromagnetic wave, homogenous or nonhomogenous, lossy or unlossy medium), the farfield analytical results are obtained by means of the saddle point method or its variational option such as the stationary phase method or the steepest descent-path method. Each of them requires evaluation of the singularities, branch points and branch cuts, as the integrand always contains the reflection coefficient $V(\theta)$ (Eq. (1)) with a square root term, thus it is a multivalued function determined on the Riemann surface. Performing integration, the adequate contour of integration must begin and end on the same leaf of Riemann's surface, so it should not cut the branch line or cut it twice. The branch cuts start at branch points $\pm \theta_{cr}$ (Fig. 2).

Applying the saddle point method, one comes to the result that the potential of the reflected wave can be expressed symbolically in the form [1]

$$\Phi_{\rm ref}(R,\alpha) = V(\alpha)\frac{e^{ikR}}{R} + \int_{L} F(\theta)d\theta,$$
(15)



Fig. 2. Contour of integration for the reflected field in the complex θ plane for the angles of incidence $\alpha < \theta_{cr}$ (left) and $\alpha > \theta_{cr}$ (right). Integration around the branch cut (line L) represents the lateral wave appearing above the critical angle.

where the second component represents the lateral wave and is equal to zero if the angle of incidence is less than the critical angle. Due to the symmetry properties, the integral over L, representing the lateral wave, can be written down as

$$\Phi_{\rm lat}(R,\alpha) = \int_{L}^{F(\theta)} d\theta = c \int_{\theta_{cr}}^{i\infty} e^{ikR\cos(\theta-\alpha)} \frac{4m\cos\theta\sqrt{n^2 - \sin^2\theta}}{(m\cos\theta)^2 - (n^2 - \sin^2\theta)} \sqrt{\sin\theta} d\theta.$$
(16)

To sum up, for angles of incidence $\alpha > \theta_{cr}$, the potential of the reflected wave can be represented by a sum of two waves – the wave reflected according to Snell's law, and the lateral wave:

$$\Phi_{\rm ref} = \Phi_{\rm snell} + \Phi_{\rm lat}, \qquad \alpha > \theta_{cr}.$$
(17)

Geometrical representations of a lateral wave, acoustic and electromagnetic, are given in Figs. 3 and 4.



Fig. 3. Schematic representation of Snell's (1) and lateral (2) acoustic waves on a plane boundary between two homogenous media for the reflection coefficient n < 1.



Fig. 4. Detection of lateral electromagnetic wave originated from the source S located at the depth z_0 in the earth or ocean.

Detailed calculations lead to the expression for the potential of the spherical wave [1]

$$\Phi_{\text{lat}} = \frac{2in}{k_1 m (n^2 - 1)\sqrt{R}L_1^{3/2}} e^{ik_1(L_0 + L) + ik_2 L_1},$$
(18)

where k_1, k_2 are wave numbers in both media. If the receiver is situated on the boundary between two media or close to $(L \approx 0)$ and moreover $L_1 \gg L_0$, then $R \approx L_0$ and thus

$$|\Phi_{\rm lat}| = \frac{2n}{k_1 m (n^2 - 1) R^2},\tag{19}$$

what means that the potential of the lateral wave decreases with the square of distance from the source. For the electromagnetic wave (Fig. 4) the last formula takes the form

$$|\Phi_{\rm lat}| = \frac{2n_0^2}{k_0(1-n_0^2)R^2} e^{-k_0\delta z_0},\tag{20}$$

where δ is the damping coefficient of the ground, k_0 is the wave number in the air and $n_0 = c_2/c_1 = 1/n$ (because the second medium is air, $c_2 = c_0$) and z_0 is the depth on which the source is located ($z_0 \approx L_0$, especially for long distances L). It is obvious that the electromagnetic lateral wave reaching the receiver is much stronger than the direct wave because of significant wave absorption in the ground or water [1].

The author of [10] expects that there should also appear some electromagnetic precursors waves, generated some time before the earthquake takes place. Damped dipole oscillations occurring due to some electrical properties of rocks in the lithosphere are considered to be responsible for excitation of extra-low and ultra-low frequency (ELF, ULF) waves.

6. Lateral wave in lossy environment

In some problems, as for example during propagation of radio waves in a forest or sound waves in air, when long distances are considered, the attenuation of wave amplitude with the distance cannot be ignored. The property of energy dissipation is expressed by complex wave number, which in turn demands the complex wave propagation velocity, refraction coefficient, electrical permittivity and magnetical permeability.

Introducing the wave number as a complex number $\tilde{k} = k + ik'$, the wave propagating along the x-axis can be written as $A_0e^{i(\tilde{k}x-\omega t)} = A_0e^{-k'x}e^{i(kx-\omega t)}$, thus $A_0 \exp(-k'x)$ represents the decaying amplitude, with $k' = \delta$ being the absorption coefficient. The basic relation, $\tilde{k} = \omega/\tilde{c}$, leads to complex velocity, $\tilde{c} = c + ic'$. For a given absorption coefficient δ , bearing in mind that $\delta \to 0$ results in $c' \to 0$ and $\tilde{c} \to c$ (c is understood as propagation velocity /phase velocity in a considered medium)

$$\operatorname{Im} c = c' = \frac{-\omega + \sqrt{\omega^2 - 4\delta^2 c^2}}{2\delta},\tag{21}$$

where the sign "+" between two terms in the numerator was chosen according to what was stated above. The real and imaginary parts of the refraction coefficient, $\tilde{n} = n + n'$, for the wave originating from medium 1 and passing to medium 2, $\tilde{n} = \tilde{c}_1/\tilde{c}_2$, are calculated to be

Re
$$\tilde{n} = n = \frac{c_1 c_2 + c_1' c_2'}{(c_2)^2 + (c_2')^2} \cong \frac{c_1 c_2}{|\tilde{c}_2|^2} = \frac{\text{Re } \tilde{c}_1 \text{Re } \tilde{c}_2}{|\tilde{c}_2|^2}, \text{ if } c_2' \to 0,$$
 (22)

Im
$$\tilde{n} = n' = \frac{c_1'c_2 + c_1c_2'}{(c_2)^2 + (c_2')^2} \cong \frac{c_1'c_2}{|\tilde{c}_2|^2} = \frac{\operatorname{Re} \tilde{c}_2\operatorname{Im} \tilde{c}_1}{|\tilde{c}_2|^2}, \quad \text{if} \quad c_2' \to 0.$$
 (23)

In many applications, the wave absorption in the second medium is negligible $(c'_2 \rightarrow 0)$, what is reflected in approximate terms in the last two expressions.

For electromagnetic waves and lossy media the reflection coefficient is equal to $\tilde{n} = \sqrt{\tilde{\varepsilon}_2 \tilde{\mu}_2 / \tilde{\varepsilon}_1 \tilde{\mu}_1}$, where, in general, electrical permittivity and magnetical permeability are complex.

For media of similar magnetic permeability ($\tilde{\mu}_1 \approx \tilde{\mu}_2$), we have $\tilde{n} = \sqrt{\tilde{\epsilon}_2/\tilde{\epsilon}_1}$, so if the second medium is air ($\epsilon_2 = \epsilon_0$), then $\tilde{n} = 1/\sqrt{\tilde{\epsilon}_{1r}}$, where $\tilde{\epsilon}_{1r}$ denotes relative electrical permittivity of the first medium. In forest environment, important in many applications, its value in a canopy medium (branches and leaves) is equal to: $\epsilon_{1r} = 1.03$ and $0.006 < \epsilon'_{1r} < 0.06$.

7. Lateral wave in radio-communication

Progress in wireless technology and continuous demand for more and more efficient and reliable devices result in many new applications connected with extension of the frequency band towards high frequency (HF), very high frequency (VHF), up to ultra-high (UHF). This calls for a more precise diffraction model accounting for those attributes of environment, which may no longer be neglected with decreasing wavelength. Since ground, vegetation or air layers constitute the usual wave propagation medium, the model should account for possible reflections and refraction, multipath propagation, scattering on nonplanar boundary between two media and their inhomogeneity. All these assumptions severely complicate the mathematical formulae thus, in practice, only some of them are taken into account, according to specific future applications.

Results of measurements carried out several decades ago, for the emitter and receiver placed in forest, depicted a considerably smaller wave attenuation than that predicted by the theory of that time. The distortion was especially visible for long distances between the emitter and receiver. The first to propose an explanation was TAMIR [11] who pointed out at a new possible wave, apart from direct and reflected wave, to reach the receiver – the lateral wave. Ray theory describes it by means of a ray emitted from the dipole source in the forest at the critical angle, travelling in air along the flat interface with adequate velocity and negligible attenuation to emerge, in turn, at the receiver at the critical angle. Considering planar interface between two homogenous media representing forest and air and accounting for wave attenuation in the forest, Tamir recognised the lateral wave even as a dominant wave detected by a distant receiver.

As it was said before, to meet the requirements of modern technique, more sophisticated models of diffraction are considered, assuming medium inhomogenity, roughness of the boundary between the media etc. [12–13]. As most of the surface on Earth is covered with water (seas, oceans) or plants (fields, forests), the wave propagation in a stratified medium, accounting for phenomena on the media interface (water/sediment, air/water, air/ground, air/plants) are of substantial interest, especially if in one medium the waves experience visible attenuation. The situation often encountered in radiocommunication is the one considered some forty years ago by TAMIR [11], with the emitter and receiver embedded in a forest, anyhow the frequencies applied nowadays belong to the VHF or UHF band (200-2000 MHz), what demands more complex modelling. First, four layers should be distinguished: ground, trunks, canopy, constituted by branches and leaves, and air, each one characterised by its electrical permittivity $\tilde{\varepsilon}$ being in general, complex. The magnetic permeability of all these four layers can be assumed to be equal to vacuum permeability μ_0 . Due to the wavelength, the trunk and branches and leaves layer should be considered to be anisotropic, with dyadic permittivity. The dipole source of electrical momentum **p**, with current density

$$\mathbf{j}\left(\mathbf{r}'\right) = \mathbf{p}\delta\left(\mathbf{r}' - \mathbf{r}_0\right),\tag{24}$$

where $\delta()$ means the Dirac distribution, is located at the point \mathbf{r}_0 in the trunk layer.

Two methods are usually applied to solve the problem – the Hertz vectorial potential (Eq. (8)) or the Green's function technique, which leads to the following result for the electric field:

$$\mathbf{E}_{i}\left(\mathbf{r}\right) = i\omega\mu_{0}\int_{V}^{\prime}\overline{\mathbf{G}}\left(\mathbf{r},\mathbf{r}^{\prime}\right)\mathbf{j}\left(\mathbf{r}^{\prime}\right) \, dV^{\prime},\tag{25}$$

where integration is taken over volume V', $\overline{\mathbf{G}}$ denotes dyadic Green function reflecting medium anisotropy, subscript i – consecutive layers. Applying the adequate expressions for the reflection coefficient (1) to the vertical and horizontal term of the dipole

momentum \mathbf{p} and adopting the saddle point method, which is especially suitable for high frequencies, the approximate evaluation of the electric field is obtained.



Fig. 5. Propagation of radio waves through forest in the case of emitter and receiver located in vegetation layers (trunks or branches and leaves), when strong lateral wave (continuous line) accompanies the reflected and refracted waves (dashed lines). The lateral wave would not contribute to the field at the observation point located in the air, since it is an evanescent wave there and it decays exponentially.

Even though the method is approximate, it provides a better physical insight into the considered phenomena than the exact solution [6], since different terms resulting from integration over the singularities or branch cuts could be ascribed to different components of the field at the receiver, such as direct wave, waves reflected at the interfaces and lateral wave, the latter being represented by integration over the branch-cut and thus appearing for the angle of incidence being not smaller than the critical angle (compare Fig. 2).

8. Some other applications

Another contemporary application of the lateral wave phenomena is the problem of detection of underground objects, mines in between, [14] by means of ground penetrating radar [15], located on media interfaces. Objects of interest are often situated in the near–field, within a few wavelengths, when assumptions of the geometric optics fail and interference of the appearing space (direct) and lateral waves have to be considered. The theoretical model can be investigated by means of numerical methods such as the finite difference time-domain method. Directivity patterns, theoretical and measurement, indicate a strong interference of both waves (compare Fig. 6).

The far-field criteria of dipole propagation in the absence of interface are fairly inadequate to provide acceptable explanation of the results obtained. The appearance of the lateral wave must be considered, otherwise a significant discrepancy between predictions of the theory and the experimental data would appear and the hidden underground targets would not be found.



Fig. 6. Wave fronts of direct (dashed line) and lateral (continuous line) waves for refraction coefficient n = 0.25 and the source located at the media interface. Strong interference between both waves is observed.

In medicine, the lateral wave was observed in propagation of ultrasonic waves in human cortical bones, when the so-called axial transmission technique had been applied [16–17]. The measurements were carried out at the frequency 1.25 MHz, which corresponds to 0.003m waves in bone tissue. It has been determined that the first arriving signal is the lateral wave propagating along the cortical bone. The arriving signal analysis provides us with information on the state of the bone and is important in osteoporosis diagnostics.

9. Conclusions

The essential role played by the lateral wave in various phenomena is based on its two features:

• It travels faster and reaches the receiver first. For that reason, in seismology the lateral sound wave is called P-wave, what means Primary wave (but also Pushwave). It is a longitudinal wave and is heard and felt as a sharp thud. Earthquakes are accompanied also by transverse wave, called S-wave, what means Secondary wave. From the time difference between the arrival of P and S-wave, the distance from the earthquake centre can be estimated.

The same method can be applied to electromagnetic waves – adequate network of radio antennas would allow to locate the earthquake.

- It plays the decisive role in the case of radio-communication between points situated in the strongly absorbing media, such as ground or ocean. The direct and reflected electromagnetic waves propagate then in absorbing medium and are rapidly damped. As a result, at great distances, only the lateral wave remains. Apart from that, some other features are important in lateral wave phenomena.
- It is an evanescent wave and it decays exponentially away from the interface in the medium in which it travels faster.
- It appears also at the nonplanar interface, but due to scattering it is weaker.
- In some problems it complicates the directivity patterns producing lobes due to interference and blurring the simple geometric optics approach.

References

- [1] BREKHOVSKIKH L. M., Waves in layered media, Academic Press, New York 1980.
- [2] BREKHOVSKIKH L. M., The reflection of spherical waves at a plane interface between two media, Journal of Technical Physics, 18, 4, 455–482 (1948).
- [3] SOMMERFELD A., Über die Ausbreitung der Wellen in der drahtlosen Telegraphie, Ann. Physik, 28, 665–736, 1909.
- [4] WEIL H., Ausbreitung elektromagnetischer Wellen über einem ebenen Leiter, Ann. Physik, 60, 481, 1919.
- [5] SNAKOWSKA A., WITKOWSKI P., IDCZAK H., The saddle point method applied to some problems in acoustics, Proc. of Int. Symp. on Hydroacoustics and Ultrasonics, Jurata, 321–326, 1997.
- [6] SNAKOWSKA A., IDCZAK H., *The saddle point method applied to selected problems in acoustics*, submitted to Acustica.
- [7] WESTWOOD E. K., Complex ray methods for acoustic interaction at a fluid-fluid interface, J. Acoust. Soc. Am., 85, 4, 1872–1884 (1989).
- [8] MORSE P., FESHBACH H., Methods of theoretical physics, Mc.Graw-Hill, London 1953.
- [9] INGARDEN R. S., *Classical and quantum electrodynamics* [in Polish], Skrypty Uniw. Mikołaja Kopernika, Toruń 1970.
- [10] MOGNASCHI E. R., On the possible origin, propagation and detectability of electro-magnetic precursors of earthquake, Atti Ticinensidi Scienze della Terra, 43, 111–118 (2002).
- [11] TAMIR T., On radio-wave propagation in forest environment, IEEE Trans., Antennas Propagat., AP-15, 806–817 (1967).
- [12] SARABANDI K., KOH I. S., Effect of canopy air interface roughness on HF VHF wave propagation in forest, IEEE Trans. Antennas Propagat., 50, 2, 111–121 (2002).
- [13] LI L. W., LEE C. K., YEO T. S., Wave mode and path characteristics in a four-layered anisotropic forest environment, IEEE Trans. Antennas Propagat., 52,8, 2445–2455 (2004).
- [14] LE PAGE K. D., SCHMIDT H., Bistatic synthetic aperture imaging of proud and buried targets from an autonomous underwater vehicles, IEEE Journal of Oceanic Engineering, 27, 3, 471–483 (2002).
- [15] RADZEVICIUS S. J., DANIELS J. J., CHEN C., C., GPR H-plane antenna patterns for horizontal dipole on a half-space interface, The Eigth International GPR Conference, SPIE vol. 4084, Gold Coast, Australia, May, 2000.
- [16] MOILANEN P., Ultrasonic guided wave measurements in bone (PhD. Thesis), Research Report 8/2004, Department of Physics, University of Jyvaskyla.
- [17] CAMUS E., TALMANT M., BERGER G., LAUGIER P., Analysis of the axial transmission technique for the assessment of skeletal status, J. Acust. Soc Amer., 2000.