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A NOTE ON IDENTIFYING A TRANSMISSION FACTOR IN ROOM ACOUSTICS

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In this study, as a concrete example of the non-linear inverse problem in indoor sound fields, a complex sound fluctuation leaked through an aperture gap in the partition door of a coupled room under the actual sound environment has been discussed. A probability evaluation theory introducing a simple stochastic inverse system model has been proposed first especially from an object-oriented viewpoint. More concretely, in order to evaluate the output response probability distribution of the acoustic intensity in the sound receiving room, after introducing time-averaged stochastic variables, we have identified functionally the leakage property as some variability of a transmission factor. Then, we have predicted the output response probability distribution for a new arbitrary stochastic input once after establishing a prediction theory for the probability distribution of a multiplicative model. Finally, an effectiveness of the proposed theory has been experimentally confirmed too by an application to actual indoor sound field data.

Key words: response probability prediction, inverse system model, room acoustics.

1. Introduction

For inverse problems in indoor sound fields, there are some representative examples such as an estimation of the intensity of sound source using the acoustic intensity observed at a sound receiving point, high faithful reproduction of the acoustic environment and so on. In recent years, many research results have been reported in an equalization (deconvolution) problem reproducing the sound source signal by constructing functionally the inverse system of the transmission path [1]. Many of them tend to discuss the artificial side of the methodology more richly rather than variety of the fluctuating phenomenon and problem of the evaluation. For example, the well-known AR model and ARMA model are essentially introduced by the assumption of the linearity of the system in advance, and/or adopt the approximation of the more simple blind equalization (blind deconvolution) problem using the least squares method under the assumption of the minimum phase transition in advance. However, even in the evaluation purpose of the equalization, it must be noticed that not only average behavior but also skirts information of the fluctuation of the observed data should be sometimes utilized because the fluctuation of the actual environmental phenomenon often shows non-Gaussian, nonlinear and unsteady distribution shapes.

In this paper, as one specific example of the inverse problem in indoor sound fields, an evaluation problem of fluctuation of complicated living environmental noises penetrating through an aperture gap of the partition door of a coupled room has been discussed. To begin with, it is nonlinear system, but an equalization system model which is simplified as much as possible is introduced first. Then, a kind of a probability evaluation theory introducing a practical inverse system model to the input is newly proposed in order to predict the output response fluctuation distribution from the object-oriented viewpoint. Concretely, in order to approximately overcome unavoidable causality in the inverse problem from an output to the input and filtering function throughout each point of time process, the leakage from the aperture gap is functionally equalized in the form of inverse system model by using the acoustic intensity matched to the evaluation purpose (acoustic intensity is the average taken throughout for the multipoint of time process). Then, against to this leakage sound field in applying the new sound input, the output response fluctuation distribution is predicted once after the establishment of the distribution transition theory based on the Mellin transformation type characteristic function of the multiplication model. Finally, by applying to some complicated indoor sound fields under the actual environment, a part of the effectiveness of the theory is also experimentally confirmed.

2. General theory

2.1. Stochastic inverse system model matched to prediction of response fluctuation probability distribution

Now, let us pay our attention to the complicated real coupled room partitioned by the sound insulation door. Let x be the acoustic intensity in the sound generating room and y the acoustic intensity penetrating through an aperture gap of the partition door. In the actual living environment, even if the acoustic intensity in the sound generating room is fixed, the acoustic intensity y observed in the sound receiving room shows a complicated fluctuation pattern caused by varieties of the transmission path and nonuniformity of the sound field of sound receiving room itself. Here, under introducing a very simplified model that the transmission factor fluctuates functionally in appearance if taking the skirts part of fluctuation pattern into consideration, let w be the transmission factor functionally equalized to the output fluctuation form as random variable in the inverse problem. And if the fact that the sound in the sound receiving room can not be observed either when sound input is not applied or when w is zero is taken into consideration, the acoustic intensity y observed in the sound receiving room is rationally given using the very simplified multiplication model:

$$y = wx. \tag{1}$$

If there is a purely physical sound insulation system under the ideal environment and if only an averaged style on the intensity scale by neglecting the skirts part of fluctuation, Eq. (1) can be called a linear model because w is considered to have a fixed value. Under such an ideal situation, of course, w means the physical transmission factor only in an average image. But w shows the random fluctuation in the actual environment because the dependence to input x exists owing to the actual nonlinearity (e.g., closely related to the skirts part of fluctuation). As a result, not only a linear correlation but also a nonlinear correlation latently exists between input x and output y. Moreover, because it is a main purpose of this inverse problem to predict the fluctuation distribution of the output y with the arbitrary new input x, the transmission factor w in Eq. (1) must be functionally identified using x and y taken as the past learning data in advance. Consequently, it becomes a problem to solve the inverse system model for this nonlinear system, as follows:

$$w = \frac{y}{x}. \tag{2}$$

However, when Eq. (2) is solved, in the conventional approach which introduces a least square norm from the methodology-first viewpoint into the evaluation, it is clear to be very difficult from the purpose-first and/or phenomenon-first viewpoint. For the specific problem solving under such a standpoint, the systematic technique seems not to be established yet.

In this study, as mentioned above, some new analysis method not using the physical transmission factor w but using an apparently functional transmission factor w considering the fluctuation has been introduced. Concretely, from the input x and the output y observed simultaneously on intensity scales, the transmission factor w in Eq. (2) should be firstly learned functionally as the fluctuation distribution. Then, according to the inverse model of the Eq. (1), the fluctuation probability distribution of y under another kind of input x can be predicted.

2.2. Prediction of fluctuation probability distribution of transmission sound y for arbitrary sound input x

Especially, the Mellin transformation type characteristic function is noticed in order to treat the probability distribution function (abbr. PDF) of non-negative random variable matched to the multiplication model. The Mellin transformation type characteristic function of the transmission sound y in Eq. (1) is given, as follows:

$$
\begin{aligned}
M_y(r) &\equiv \int_0^\infty y^{r-1} P_y(y)\, dy \\
&= \int_0^\infty \int_0^\infty (w \cdot x)^{r-1} P_{w \cdot x}(w,\, x)\, dw dx.
\end{aligned} \tag{3}
$$

Here, for the PDF $P_y(y)$ of y, the series expansion type expression based on the PDF of random variable w and/or x has been already proposed [2]. The case based on the PDF $P_x(x)$ of x is shown, as follows:

$$
P_y(y) = \sum_{n=0}^\infty \frac{(-1)^n}{n!} \left(\frac{d}{dx} x \cdot \right)^n \langle (\ln w)^n | x \rangle\, P_x(x) \bigg|_{x \to y} . \tag{4}
$$

Here, of course the first expansion term ($n = 0$) of $P_y(y)$ agrees with the $P_x(x)$ adopted as the basic distribution. Then, by increasing the number of expansion terms, the compensation is made hierarchically by the higher order moments of the random variable w. Therefore, there is the simplicity that only moment information may be used after the second expansion term. However, a large number of expansion terms must be adopted for the convergence of estimated distribution, when the basic distribution is not so dominant.

Now, first of all, if the special case when w and x are statistically independent of each other is assumed in the basis of the analysis and each follows gamma distribution (matched to the non-negative random variable), $P_w(w)$ and $P_x(x)$ can be defined, respectively as follows:

$$
P_w(w) = \frac{w^{m_w - 1} e^{-w/S_w}}{\Gamma(m_w) S_w^{m_w}} , \tag{5}
$$

$$
P_x(x) = \frac{x^{m_x - 1} e^{-x/S_x}}{\Gamma(m_x) S_x^{m_x}} , \tag{6}
$$

where, m_w, S_w and m_x, S_x denote the parameters of the gamma distribution of w and x, respectively. And, the Mellin transformation type characteristic function [3] of w and x is given, respectively as follows:

$$
M_w(r) = \frac{\Gamma(m_w + r - 1) S_w^{r-1}}{\Gamma(m_w)} , \tag{7}
$$

$$
M_x(r) = \frac{\Gamma(m_x + r - 1) S_x^{r-1}}{\Gamma(m_x)} . \tag{8}
$$

Therefore, from Eqs. (3), (7) and (8), the Mellin transformation type characteristic function of y can be expressed, as follows:

$$M_y(r) = \frac{\Gamma(m_w + r - 1)\Gamma(m_x + r - 1)}{\Gamma(m_w)\Gamma(m_x)}(S_w S_x)^{r-1}. \tag{9}$$

Next, let us apply the following formula [4]:

$$\int_0^\infty \xi^\mu K_\nu(a\xi)\, d\xi = 2^{\mu-1}a^{-\mu-1}\Gamma\left(\frac{1+\mu+\nu}{2}\right)\Gamma\left(\frac{1+\mu-\nu}{2}\right), \tag{10}$$

where, $K_\nu(\cdot)$ is a Kelvin function (:the second kind) as one of modified Bessel functions. Equation (10) can be rewritten using specific parameters: $\mu = 2(r-1) + m_w + m_x - 1$, $a = 2/\sqrt{S_w S_x}$ and $\nu = m_w - m_x$, as follows:

$$\begin{aligned}
&\int_0^\infty \xi^{2(r-1)+m_w+m_x-1} K_{m_w-m_x}\left(\frac{2}{\sqrt{S_w S_x}}\xi\right) d\xi \\
&= 2^{2(r-1)+m_w+m_x-2}\left(\frac{2}{\sqrt{S_w S_x}}\right)^{-\{2(r-1)+m_w+m_x\}} \\
&\qquad \cdot\Gamma\left(\frac{2(r-1)+2m_w}{2}\right)\Gamma\left(\frac{2(r-1)+2m_x}{2}\right) \\
&= 2^{-2}(S_w S_x)^{(m_w+m_x)/2}\Gamma(m_w+r-1)\Gamma(m_x+r-1)(S_w S_x)^{r-1}.
\end{aligned} \tag{11}$$

Therefore, from Eqs. (9) and (11), $M_y(r)$ can be expressed as follows:

$$\begin{aligned}
M_y(r) &= \frac{4}{\Gamma(m_w)\Gamma(m_x)(S_w S_x)^{(m_w+m_x)/2}} \\
&\qquad \int_0^\infty \xi^{2(r-1)+m_w+m_x-1} K_{m_w-m_x}\left(\frac{2}{\sqrt{S_w S_x}}\xi\right) d\xi.
\end{aligned} \tag{12}$$

In the right-hand side of the Eq. (12), when the measure preserving transformation of the probability is performed as $y = \xi^2$, the following expansion can be obtained:

$$M_y(r) = \int_0^\infty y^{r-1}\frac{2y^{(m_w+m_x)/2-1}K_{m_w-m_x}\left(2\sqrt{\frac{y}{S_w S_x}}\right)}{\Gamma(m_w)\Gamma(m_x)(S_w S_x)^{(m_w+m_x)/2}}\, dy. \tag{13}$$

Because Eq. (13) holds true at every values of r, by the collation of Eq. (3) with Eq. (13), $Py(y)$ is given, as follows:

$$P_y(y) = \frac{2y^{(m_w+m_x)/2-1}K_{m_w-m_x}\left(2\sqrt{\frac{y}{S_w S_x}}\right)}{\Gamma(m_w)\Gamma(m_x)(S_w S_x)^{(m_w+m_x)/2}}. \tag{14}$$

Furthermore, in order to apply this theory to the case when various kinds of linear and nonlinear correlation exist between w and x and/or the case when each distribution of w and x deviated from gamma distribution, $Py(y)$ is derived basing on the statistical Laguerre series expansion type probability expression [5]:

$$P_y(y) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} C_{mn}\left(\frac{\partial}{\partial S_w}\right)^m \left(\frac{\partial}{\partial S_x}\right)^n \cdot \frac{2y^{(m_w+m_x)/2-1} K_{m_w-m_x}\left(2\sqrt{\dfrac{y}{S_w S_x}}\right)}{\Gamma(m_w)\Gamma(m_x)(S_w S_x)^{(m_w+m_x)/2}}, \tag{15}$$

where, C_{mn} denotes the expansion coefficients reflecting not only lower but also higher order correlations between w and x, as follows:

$$C_{mn} = (-1)^{m+n}\frac{\Gamma(m_w)\Gamma(m_x)S_w^m S_x^n}{\Gamma(m_w+m)\Gamma(m_x+n)} \left\langle \mathrm{L}_m^{(m_w-1)}\left(\frac{w}{S_w}\right)\mathrm{L}_n^{(m_x-1)}\left(\frac{x}{S_x}\right)\right\rangle. \tag{16}$$

3. Experiment

In order to confirm a part of the practical effectiveness of the proposed theory mentioned above, some principle experiment on the complicated living sound environment penetrating through an aperture gap of the partition door of the coupled room was carried out.

3.1. Measurement situation

Two reverberation chambers partitioned by the sound insulation door were used as a coupled room. The door was opened during the experiment at an aperture gap of 30 mm. A white noise of the 1/3 octave band with a center frequency of 200 Hz was applied through the loudspeaker in the sound generating room. In the sound generating room and sound receiving room, the observed sound waves were recorded on a level recorder through each microphone, respectively. Input-output data were converted into digital data from analog at a sampling period of 1 second and a quantization level of 12 bit. The 1,000 data was simultaneously sampled, respectively.

3.2. Distribution estimation of functionally equalized transmission factor w

Basing on the former 500 data points, the stochastic inverse system model of Eq. (2) was functionally solved. The fluctuation distribution of the functionally equalized transmission factor w is shown in Fig. 1. From this figure, it is clear that it is very difficult in such a complicated case of this study to grasp the fluctuation of the phenomenon only in the form of an averaged transmission factor of physical quantity (constant parameter), as is generally adopted in the standard sound-insulating wall under the ideal sound environment. Furthermore, it can be found that the fluctuation distribution of the functionally equalized transmission factor is approximated by the gamma distribution (owing to its non-negative property).

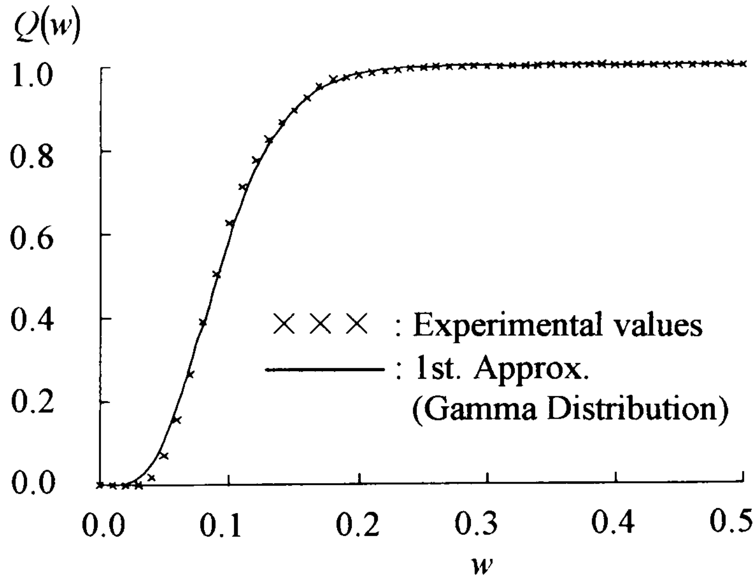

Fig. 1. Comparison between theoretically fitted curve and actually observed values for fluctuation probability distribution of functionally equalized transmission factor w.

3.3. Prediction of response probability distribution for arbitrarily fluctuating sound input

Next, basing on the latter 500 data points, as a case of applying another kind of sound input wave x in the sound generating room, the fluctuation probability distribution of observed sound wave y in the sound receiving room was predicted. That is to say, by using fluctuation distribution information of w in the former section and correlation information from the lower order to the higher orders between x and w, the distribution of y was predicted basing on the latter data of (different kind of) x with the theoretical Eq. (15). A comparison of the theoretical distribution with the experimentally observed distribution is shown in Fig. 2. In this figure, though the theoretical curve only in the first expansion term ($m + n = 0$) can not explain except for near the average, it can be found that the theoretical curve explains the observation data well especially in the skirt part of the distribution, if the higher order correlation information between x and w is taken into consideration more and more by increasing the expansion term.

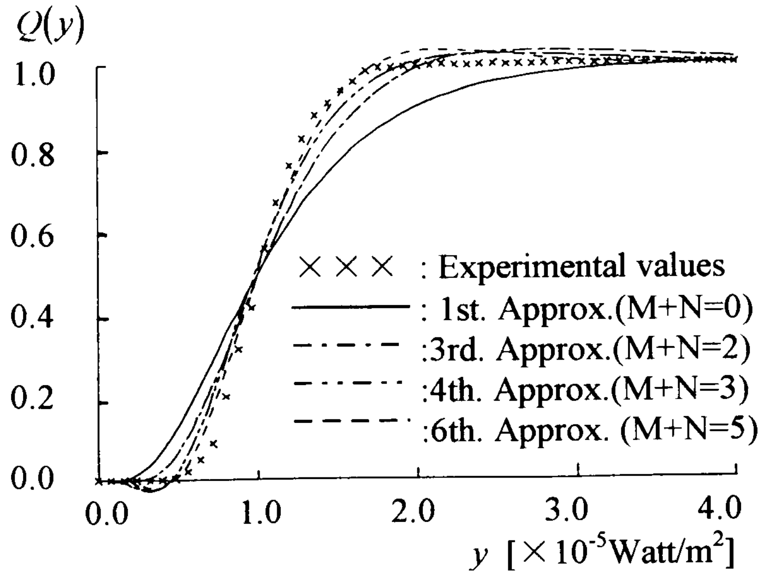

Fig. 2. Comparison between theoretically predicted curve and true values for the specific output response probability distribution.

4. Conclusions

In this paper, a specific example of the nonlinear inverse problem in indoor sound field under the actual environment was noticed. That is to say, focusing on the complicated living sound environment penetrating through an aperture gap of the partition door of the coupled room, once after introducing the practically simplified inverse system model matched to a prediction of the response fluctuation distribution especially from the object-oriented viewpoint, a probability evaluation theory which took the various types of input dependence of the system into consideration was newly proposed. Concretely, in order to overcome practically unavoidable causality in the inverse problem from an output to the input and filtering function throughout each point of time process, the leakage from the aperture gap was functionally equalized as the inverse system model by using the acoustic intensity matched to the evaluation purpose. Then, for this actual leakage sound field with some new sound stochastic input, the output response fluctuation distribution was predicted under establishment of probability distribution transition theory especially based on the Mellin transformation type characteristic function of the multiplication model. Finally, by applying to a complicated indoor sound field under the actual environment, a part of the effectiveness of the theory was also experimentally confirmed.

Since this study is at an initial stage of the research, there still remain many future problems ought to be piled up in addition to this basic research. For example, i) an application to nonlinear system under the other actual environment, ii) a simplification of the theory aiming at the practicability, iii) an expansion of the theory for the actual case when background noise exists, iv) an establishment of the systematic technique reflecting the existence way of the fluctuation without viewing inverse system deterministically and so on.

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