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ULTRASONIC WAVES IN DENSIFIED SUSPENSIONS

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A theory of propagation of ultrasonic waves in sediments is developed. The formulae for the phase velocity and the attenuation coefficient are determined as functions of wave frequency and the mass fraction of the solid phase. These formulae can be used, after suitable calibration, for determination of the solid mass fraction or the water content in densified suspensions. These structure parameters can be determined by measuring the transition time of ultrasonic wave through a given distance of sediment. The phase velocity dispersion curves and the attenuation coefficients determined theoretically and experimentally are plotted as functions of the solid mass fraction for sediment.

Key words: ultrasounds, sediment, wave propagation, phase velocity, attenuation coefficient.

1. Introduction

The aim of this paper is to establish a basis for application of ultrasounds for determination of the density profiles in setting suspensions, or simply, for determination of the solid mass fraction or the moisture content in densified suspensions (sediments). For this purpose the theory of propagation of stress waves in densified suspensions is developed and the relations between the phase velocity and the mass fraction of the solid phase are determined. Such relations after suitable calibration enable determination of the mass fraction, by measurement of the transition time of ultrasonic waves through a given layer of suspension.

There are some authors who dealt with application of ultrasound for characterization of suspensions, as for example, AHUJA [2], HARKER and TEMPLE [10] – czy prace z literatury, CLEMENT *et al.* [8], LEWANDOWSKI [14], TEBBUTT and CHALLIS [21], MALCOLM and POVEY [17], SAYAN and URLICH [19]. However, all these papers differ from each other in the approach to modeling the wave propagation in suspensions. So, there is no coherence in the results obtained by individual authors. This paper does not review all papers which have been written on the subject, but gives only the essential knowledge necessary for construction of a unified theory.

The main purpose of this paper is to construct a theory of propagation of stress waves in densified suspensions, starting from the conservation equations for a two-constituent medium. The basic conservation equations are generally valid for cohesive suspensions, which occur in sewers, reservoirs, estuarine and coastal waters. The general conservation laws are specified to the nature of densified (cohesive) suspensions. The considerations concerning cohesive suspensions (sediments), are based on the theory of dynamic consolidation typical for a porous medium filled with liquid (see BIOT [6] and [7]; DERSKI and KOWALSKI, [9]).

Adopting the theory of wave propagation to passive ultrasounds, which are of weak energy, we assume that the ultrasonic waves involve only small perturbations in the suspension through which they propagate. So the wave equations may be reduced to describe only small fluctuations in the medium, and thus, to develop the formulae for the phase velocity and the attenuation coefficient as a function of the frequency of ultrasonic wave and the mass fraction of the solid phase of suspension. Moreover, the phase velocity and the attenuation coefficient were determined experimentally making use of the ultrasonic setup. The theoretical and experimental curves were plotted for comparison, and their good agreement was established. So, one can say that ultrasonic velocity measurements offer the possibly rapid, accurate and cheap assessments of mass fractions or moisture content in cohesive suspensions.

2. General conservation equations

Sediments can be considered as two-constituent media of solid particles and fluid. The following principles and assumptions form the basis of the theory proposed in this paper:

• Every place x_i in space occupied by the suspension contains in each time t simultaneously particles of two constituents of partial mass density $\rho^\alpha = \rho^{\alpha r}\phi^\alpha$, moving with velocity v_i^α relative to some frame of reference. The partial densities are defined as products of the real mass density $\rho^{\alpha r}$ and the volume fraction ϕ^α of the constituent $\alpha = \{s(\text{solid}), f(\text{fluid})\}$. The balance of mass of constituent α in local form is expressed by the equation

$$\frac{\partial \rho^\alpha}{\partial t} + \frac{\partial \rho^\alpha v_i^\alpha}{\partial x_i} = 0. \tag{1}$$

• The rate of change of α-momentum is due to the surface force represented by the α-stress tensor σ_{ij}^α and to the body force represented by external gravity $\rho^\alpha g_i$ and a force density $\hat{p}_i^\alpha$ due to interaction of constituent α with the other constituent. Hence in local form the equation of balance of α-momentum reads

$$\frac{\partial}{\partial t}\left(\rho^\alpha v_i^\alpha\right) + \frac{\partial}{\partial x_j}\left(\rho^\alpha v_i^\alpha v_j^\alpha - \sigma_{ij}^\alpha\right) = \rho^\alpha g_i + \hat{p}_i^\alpha \qquad \text{with} \qquad \hat{p}_i^s + \hat{p}_i^f = 0. \tag{2}$$

• The rate of change of energy (internal plus kinetic energy) is due to the supply of mechanical and non-mechanical power

$$\frac{\partial}{\partial t}\left[\rho^\alpha\left(u^\alpha+\frac{1}{2}v_i^\alpha v_i^\alpha\right)\right]+\frac{\partial}{\partial x_j}\left[\rho^\alpha\left(u^\alpha+\frac{1}{2}v_i^\alpha v_i^\alpha\right)v_j^\alpha-\sigma_{ij}^\alpha v_i^\alpha+q_j^\alpha\right] \\ =\rho^\alpha g_i v_i^\alpha+\rho^\alpha r^\alpha+\hat{e}^\alpha. \qquad (3)$$

In this equation u^α is the internal energy per unit mass of constituent α, q_i^α is the heat flux, $\rho^\alpha r^\alpha$ is the volumetric energy supply (radiation), and $\hat{e}^\alpha$denotes the exchange of energy between constituents, i.e. $\hat{e}^s+\hat{e}^f=0$.

• The rate of change of entropy is due to the supply of heat and production of entropy because of irreversibility of heat and mass transport processes

$$\frac{\partial}{\partial t}\left(\rho^\alpha s^\alpha\right)+\frac{\partial}{\partial x_i}\left(\rho^\alpha s^\alpha v_i^\alpha+\frac{q_i^\alpha}{T^\alpha}\right)=\frac{\rho^\alpha r^\alpha}{T^\alpha}+\hat{s}^\alpha \qquad \text{with} \qquad \hat{e}^s+\hat{e}^f\geq 0, \quad (4)$$

where s^α is the entropy per unit mass of constituent α, T^α is the temperature, and $\hat{s}^\alpha$ denotes the density of entropy production.

These are the general equations of balance of mass, momentum, energy end entropy for the individual constituents written in local forms due to the assumption of continuous distribution of individual constituents in the sediment.

3. Constitutive equations

We assume that a sediment consists of the solid skeleton and fluid filling the pore space. The balances of mass, momentum, energy and entropy for each individual constituent lead to the Eqs. (5) to (8). We rewrite them now in a reduced form assuming additionally common temperature for both constituents:

• *balance of mass*

$$\dot{\rho}^\alpha+\rho^\alpha v_{i,i}^\alpha=0; \qquad (5)$$

• *balance of momentum*

$$\rho^\alpha\dot{v}_i^\alpha=\sigma_{ij,j}^\alpha+\rho^\alpha g_i+\hat{p}_i^\alpha; \qquad (6)$$

• *balance of energy*

$$\rho^\alpha\dot{u}^\alpha=\sigma_{ij}^\alpha v_{i,j}^\alpha-q_{i,i}^\alpha+\rho^\alpha r^\alpha-\hat{p}_i^\alpha v_i^\alpha+\hat{e}^\alpha; \qquad (7)$$

• *balance of entropy*

$$\rho^\alpha\dot{s}^\alpha+\left(\frac{q_i^\alpha}{T}\right)-\frac{\rho^\alpha r^\alpha}{T}=\hat{s}^\alpha, \qquad (8)$$

where dot over a symbol denotes the material time derivative of the individual constituent.

Eliminating the a priori defined radiation term in the balance of entropy, we can rewrite the last equation as follows:

$$-\rho^\alpha\left(\dot{f}^\alpha+s^\alpha\dot{T}^\alpha\right)+\sigma_{ij}^\alpha v_{i,j}^\alpha-\frac{q_i^\alpha}{T}T_{,i}-\hat{p}_i^\alpha v_i^\alpha+\hat{e}^\alpha=T\hat{s}^\alpha, \tag{9}$$

where $f^\alpha = u^\alpha - s^\alpha T^\alpha$ is the free energy of constituent α per unit mass.

Having in mind that the theory is developed for the propagation of ultrasonic waves in suspensions and that these waves cause only small perturbations in the medium, we can neglect the convective terms in the material derivatives of the individual constituents. Furthermore, we assume that the resistance drag of fluid is much smaller than that of solid skeleton. This justifies neglecting of the stress deviator in fluid. So, taking into account the second law of thermodynamics expressed by the last term in Eq. (8), we may write

$$-\left(\dot{F}^\alpha+S\dot{T}\right)+\sigma_{ij}^s\dot{\varepsilon}_{ij}+\sigma^f\dot{\in}-\frac{q_i^\alpha}{T}T_{,i}-\hat{p}_i^f\left(v_i^f-v_i^s\right)\geq 0, \tag{10}$$

where $\dot{F}\overset{\text{def}}{=}\rho^s\dot{f}^s+\rho^f\dot{f}^f$ – is the time derivative of total free energy density, $S=\rho^s s^s+\rho^f s^f$ – is total entropy density, $\varepsilon_{ij}=\frac{1}{2}\left(u_{i,j}+u_{j,i}\right)$ – is the strain tensor and u_i the displacement vector of the solid, $\in = U_{i,i}$ – is the volumetric strain, and U_i the displacement vector of the fluid.

Note that in the case of uniform temperature and absence of the relative fluid flow with respect to porous solid, the process can be considered to be reversible. In such a case, the Gibbs' identity of the form can be assumed

$$\dot{F}=-S\dot{T}+\sigma_{ij}^s\dot{\varepsilon}_{ij}+\sigma^f\dot{\in}. \tag{11}$$

This relation shows that the free energy for reversible processes is dependent on the temperature and strains of both the porous solid and fluid, i.e. $F = F(T, \varepsilon_{ij}, \in)$.

An irreversible process may occur because of viscous flow of fluid through the porous body. The entropy produced in the medium is transformed into heat causing an increase of temperature. This increase is shown in the relation (11), which may hold for both the reversible and irreversible processes. If it is so, then from (10) follows a residual inequality of the form

$$-\hat{p}_i^f\left(v_i^f-v_i^s\right)-\frac{q_i}{T}T_{,i}\geq 0. \tag{12}$$

It is easy to see that the sufficient (not necessary) conditions that will satisfy the inequality (12) are

$$\hat{p}_i^f=-b\left(v_i^f-v_i^s\right)\quad\text{with}\quad b\geq 0\quad\text{and}\quad q_i=-\lambda T_{,i}\quad\text{with}\quad\lambda\geq 0. \tag{13}$$

Basing on the Gibbs' identity (11), we obtain the following equations of state:

$$\begin{aligned} S &= -\left(\frac{\partial F}{\partial T}\right)\Big|_{\varepsilon_{ij},\in} = S(T,\varepsilon_{ij},\in), \\ \sigma^s_{ij} &= \left(\frac{\partial F}{\partial \varepsilon_{ij}}\right)\Big|_{T,\in} = \sigma^s_{ij}(T,\varepsilon_{ij},\in), \\ \sigma^f &= \left(\frac{\partial F}{\partial \in}\right)\Big|_{T,\varepsilon_{ij}} = \sigma^f(T,\varepsilon_{ij},\in). \end{aligned} \tag{14}$$

Developing the free energy function in Taylor's series with respect to parameters of state and using the equation of state, we arrive at the physical relations of the form (see also DERSKI and KOWALSKI [9])

$$\begin{aligned} \sigma^s_{ij} &= 2N\varepsilon_{ij} + [A\varepsilon + Q\in - \gamma^s(T-T_0)]\delta_{ij}, \\ \sigma^f &= Q\varepsilon + R\in - \gamma^f(T-T_0). \end{aligned} \tag{15}$$

According to the Biot interpretation (see Biot [6, 7]), the constants N and A correspond to the familiar Lamé coefficients in the theory of elasticity, where N represents the shear modulus of the sediment. The coefficient R represents the measure of the pressure required on the fluid to force a certain volume of the fluid into the porous body, while the total volume remains constant. The coefficient Q expresses a coupling between the volume change of the solid and that of the fluid. All these coefficients are found to be positive. Besides, the coefficients $\gamma^s = (2N+3A)\,\alpha^s$ and $\gamma^f = (2N+3A)\,\alpha^f$ can be termed the thermal moduli with α^s and α^f being the coefficients of linear thermal expansion of the solid and the fluid, respectively.

In further examination of ultrasonic waves in sediments, we shall neglect the thermal effects and the gravity forces. In such circumstances, there is no need to develop the physical relation for entropy and to construct the differential equation for temperature. So, substituting the physical relations obtained above into the balance of momentum (6), we have

$$\begin{aligned} N(u_{i,jj}+u_{j,ji}) + Au_{j,ji} + QU_{j,ji} + b(\dot{U}_i - \dot{u}_i) &= \rho^s\ddot{u}_i, \\ Qu_{j,ji} + RU_{j,ji} - b(\dot{U}_i - \dot{u}_i) &= \rho^f\ddot{U}_i. \end{aligned} \tag{16$_{1,2}$}$$

These equations have to be supplemented by the equations of mass continuity (5), namely

$$\begin{aligned} \dot{\rho}^s + \rho^s\dot{u}_{i,i} &= 0, \\ \dot{\rho}^f + \rho^f\dot{U}_{i,i} &= 0. \end{aligned} \tag{16$_{3,4}$}$$

Equations $(16)_1$ to $(16)_4$ constitute the complete system for description of wave propagation in sediments. According to BIOT [6, 7], we assume here that coefficient b is related to the coefficient of permeability k, fluid viscosity μ, and porosity ϕ by means of

$$b = \mu\frac{\phi^2}{k}. \tag{17}$$

4. Phase velocity and attenuation coefficient

In order to apply the ultrasounds for assessment of the mass fraction of the solid phase or the moisture content in a sediment, we have to determinate first of all the phase velocity. The attenuation coefficient, on the other hand, indicates the damping effects of the medium and points out whether the generated signals can propagate for a long or short distance.

We assume that the ultrasound signals are generated in one-dimensional, say x-direction. The governed functions in this case are: displacements and densities of the solid skeleton and fluid, that is

$$g = \{u_x, U_x, \rho^s, \rho^f\}. \tag{18}$$

Each of these functions is assumed to be of the form

$$g(x,t) = \overline{g} + g'(x,t), \qquad \overline{g} = \text{const} \quad \text{and} \quad \left|g'\right| << \overline{g}, \tag{19}$$

where g' denotes the local and instant fluctuation of quantity g around its equilibrium value $\overline{g}$.

The system of equations reduced to the one-dimensional case and after application of Eq. (18) reads

$$\begin{aligned} \frac{\partial^2}{\partial x^2}\Big[(2N+A)u_x^{'} + QU_x^{'}\Big] + b(\dot{U}_x^{'} - \dot{u}_x^{'}) &= \overline{\rho}^s \ddot{u}_x^{'}, \\ \frac{\partial^2}{\partial x^2}\Big[Qu_x^{'} + RU_x^{'}\Big] - b(\dot{U}_x^{'} - \dot{u}_x^{'}) &= \overline{\rho}^f \ddot{U}_x^{'}, \\ \dot{\rho}^{s'} + \overline{\rho}^s \frac{\partial \dot{u}_x^{'}}{\partial x} &= 0, \\ \dot{\rho}^{f'} + \overline{\rho}^f \frac{\partial \dot{U}_x^{'}}{\partial x} &= 0. \end{aligned} \tag{20}$$

Note that the nonlinear terms of small value are neglected in the above system of equations.

The fluctuations, which are the ultrasonic disturbances, are assumed to propagate in x-direction in the form of harmonic attenuated waves, i.e.

$$g'(x,t) = g_0 \exp i(lx - \omega t), \tag{21}$$

where ω is the frequency of waves and $l = l_r + il_i$ is a complex number consisting of the wave number l_r and the attenuation coefficient l_i.

Combining this equation with those written above and rearranging slightly, we obtain the following system of algebraic equations:

$$(a_{11}u_0 + a_{12}U_0)\, z + \beta(U_0 - u_0) - \frac{\overline{\rho}^s}{\overline{\rho}}\, u_0 = 0, \tag{22}$$

$$\begin{aligned}(a_{12}u_0 + a_{22}U_0)\, z - \beta(U_0 - u_0) - \frac{\overline{\rho}^f}{\overline{\rho}} U_0 &= 0, \\ \rho_0^s + \overline{\rho}^s (il) u_0 &= 0, \\ \rho_0^f + \overline{\rho}^f (il) U_0 &= 0.\end{aligned} \tag{22 [cont.]}$$

The following notations are introduced in these equations:

$$a_{11} = \frac{2N + A}{a}, \qquad a_{12} = \frac{Q}{a}, \qquad a_{22} = \frac{R}{a},$$
$$z = \frac{l^2}{\omega^2} c^2, \qquad c^2 = \frac{a}{\overline{\rho}}, \qquad \beta = \frac{b}{\omega \overline{\rho}},$$

and $a = 2N + A + R + 2Q$ represents the total elastic modulus and $\overline{\rho} = \overline{\rho}^s + \overline{\rho}^f$ the total mass per unit volume of the medium as a whole. Adding Eqs. $(21)_1$ and $(21)_2$ it can be seen that c represents the phase velocity in the sediment under the condition that $u_0 = U_0$ and $l_i = 0$, i.e., there is no relative motion between the constituents. The coefficient β represents the ratio of damping parameter $b/\overline{\rho}$ to the wave frequency ω. The attenuation coefficient l_i will be zero if $\beta = 0$.

The homogeneous system of Eqs. (21) has a non-trivial solution if the determinant the consisting of the coefficients standing at the wave amplitudes equals zero. Developing this determinant, we arrive at the quadratic equation of the form

$$\left(a_{11}a_{22} - a_{12}^2\right) z^2 - \frac{1}{\overline{\rho}} \left(\overline{\rho}^f a_{11} + \overline{\rho}^s a_{22}\right) z + \frac{\overline{\rho}^f \overline{\rho}^s}{\overline{\rho}^2} + i\beta\,(1 - z) = 0. \tag{23}$$

If $V = \omega l_r$ denotes the phase velocity and (l_i/l_r) – the attenuation coefficient referred to the wave number, the following two kinds of velocities and attenuation coefficients will result from the above equation:

$$\frac{V_1}{c} = \frac{\sqrt{2}}{\sqrt{\sqrt{K^2 + L^2} + K}} \quad \text{and} \quad \left(\frac{l_i}{l_r}\right)\Big|_1 = \frac{\sqrt{\sqrt{K^2 + L^2} - K}}{\sqrt{\sqrt{K^2 + L^2} + K}}, \tag{24}$$

where

$$K = \frac{1}{2}\left(p - \frac{1}{\sqrt{2}}\sqrt{\sqrt{X^2 + Y^2} + X}\right)$$
$$\text{and} \quad L = \frac{1}{2}\left(r\beta - \frac{1}{\sqrt{2}}\sqrt{\sqrt{X^2 + Y^2} - X}\right) \tag{24a}$$

and

$$\frac{V_2}{c} = \frac{\sqrt{2}}{\sqrt{\sqrt{M^2 + N^2} + M}} \quad \text{and} \quad \left(\frac{l_i}{l_r}\right)\Big|_2 = \frac{\sqrt{\sqrt{M^2 + N^2} - M}}{\sqrt{\sqrt{M^2 + N^2} + M}}, \tag{25}$$

where

$$M = \frac{1}{2}\left(p + \frac{1}{\sqrt{2}}\sqrt{\sqrt{X^2+Y^2}+X}\right)$$

$$\text{and} \quad N = \frac{1}{2}\left(r\beta + \frac{1}{\sqrt{2}}\sqrt{\sqrt{X^2+Y^2}-X}\right). \tag{25a}$$

The other notations are:

$$X = q^2 - r^2\beta^2, \qquad Y = 2r\,(p-2)\,\beta, \qquad p = r[a_{11} + (a_{22} - a_{11})\,\alpha],$$

$$q = r\sqrt{[a_{11} - (a_{22} - a_{11})\,\alpha]^2 + 4a_{12}^2\alpha\,(1-\alpha)},$$

$$r = \left(a_{11}a_{22} - a_{12}^2\right)^{-1}, \qquad \alpha = \overline{\rho}^s/\overline{\rho}.$$

Here α expresses the mass fraction of the solid phase in the sediment. Its value changes in the range $0 \le \alpha \le 1$.

We have obtained two phase velocities concerning two kinds of waves. It means that an impulse applied to the two-phase medium splits into two parts, each transported by the wave of different velocity. Such a medium has dispersive properties and the applied impulses are scattered in the medium during their propagation. Additionally, the waves are damped due to viscous properties of the fluid moving in pores.

Scattering and damping of the waves causes that waves may hardly pass through the medium. Fortunately, the ultrasonic waves are rather of high frequency. Note that high wave frequency lowers the coefficient β responsible for attenuation of the wave amplitudes.

The ratio of the phase velocity to the reference velocity c and the attenuation coefficient of the first and second kind of waves are plotted as functions of the solid mass fraction α in Figs. 1 to 4 for various combinations of material coefficients listed in Table 1.

Table 1. Possible material coefficients.

No.	$a_{11} = (2N + A)/a$	$a_{22} = R/a$	$a_{12} = Q/a$	$\beta = b/\rho\omega$
1	0.6	0.3	0.05	0
2	0.6	0.3	0.05	2.0
3	0.6	0.3	0.05	10.0
4	0.4	0.4	0.10	2.0

Analyzing the plots presented in Figs. 1 to 4 one can draw the following conclusions:

• Two kinds of longitudinal waves are induced by the ultrasonic impulses in the two-phase medium.

• The phase velocity of the first-kind wave is several times greater than the velocity of the second-kind wave, so the waves can be termed as fast and slow, respectively.

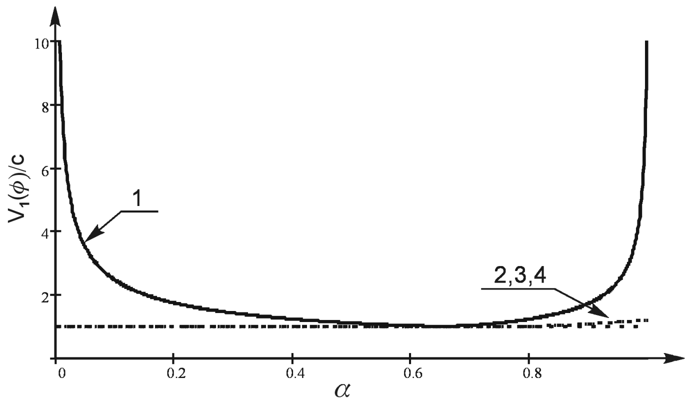

Fig. 1. Phase velocity of the first-kind wave.

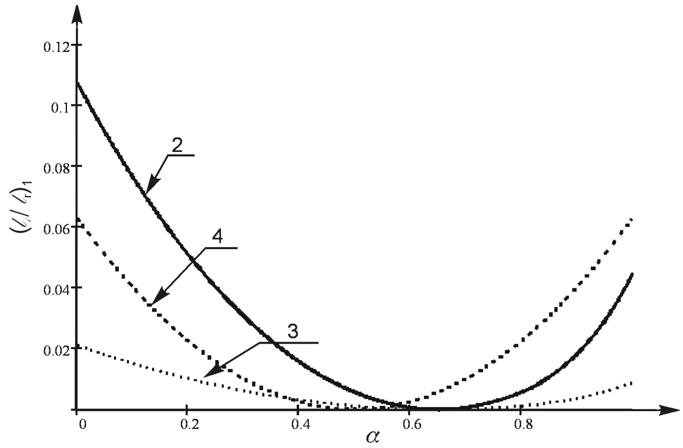

Fig. 2. Attenuation coefficient of the first-kind wave.

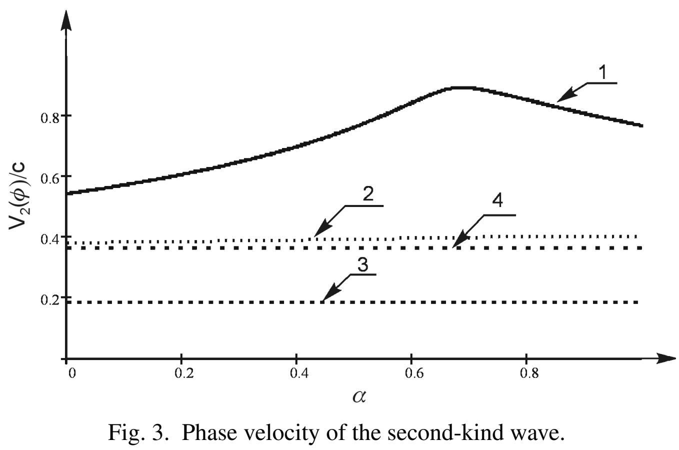

Fig. 3. Phase velocity of the second-kind wave.

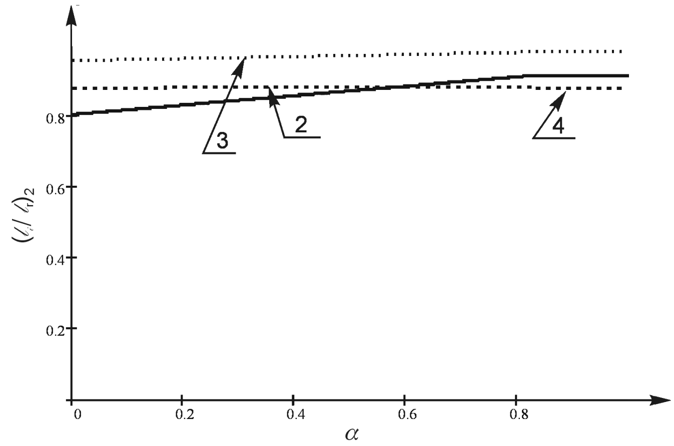

Fig. 4. Attenuation coefficient of the second-kind wave.

• The attenuation coefficient of the slow wave is, in general, much greater than that for the fast wave, particularly if the realistic range of the solid mass fraction is considered, i.e. $0.25 \leq \alpha \leq 0.8$.

• The wave velocity increases and the attenuation coefficient decreases significantly for small values of the parameter β, that is, for small damping properties of the material represented by parameter b or for very high angular frequencies ω of the waves.

Figure 5 presents the phase velocities and Fig. 6 the attenuation coefficients for the fast and slow waves as a function of β for data, which can be suitable for saturated peat of mass fraction α = 0.6, characterized by the following parameters: a_{11} = 0.5, a_{22} = 0.4, a_{12} = 0.05.

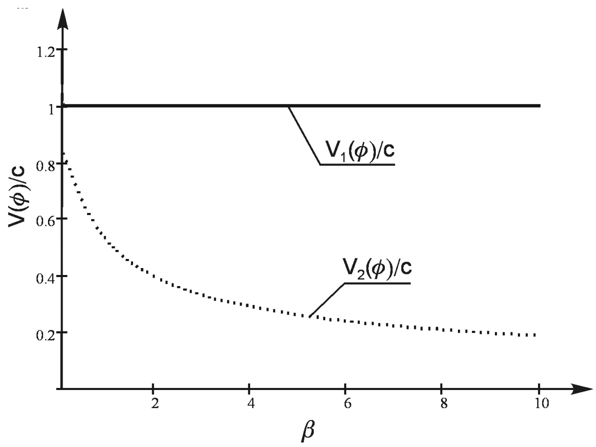

Fig. 5. Phase velocities of the fast and slow waves.

It is seen from Fig. 5 that the velocity of the fast wave is constant and equal to unity for the constant solid mass fraction $\alpha = 0.6$ independently of the β variation, and the velocity of the slow wave decreases systematically tending to zero as β increases.

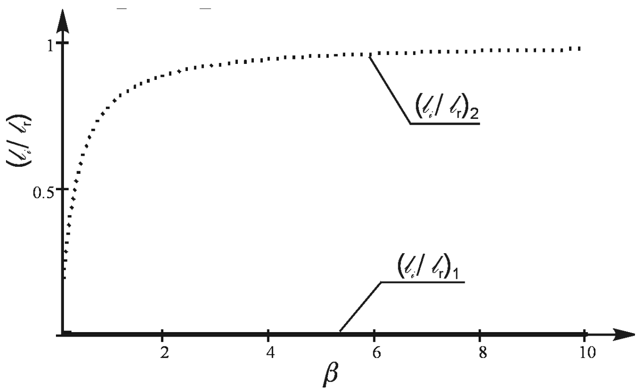

Fig. 6. Attenuation coefficients of the fast and slow waves.

The attenuation coefficient is close to zero for the fast wave and tends to unity for the slow wave as β increases. It means that the slow wave is strongly damped, and only the fast wave can be measured practically in experimental studies.

Figure 7 presents the ultrasonic tester of type UMT-01-UNIPAN, used for measurement of ultrasonic waves in peat sediment, and indirectly for measurement of the solid mass fraction of the sediment.

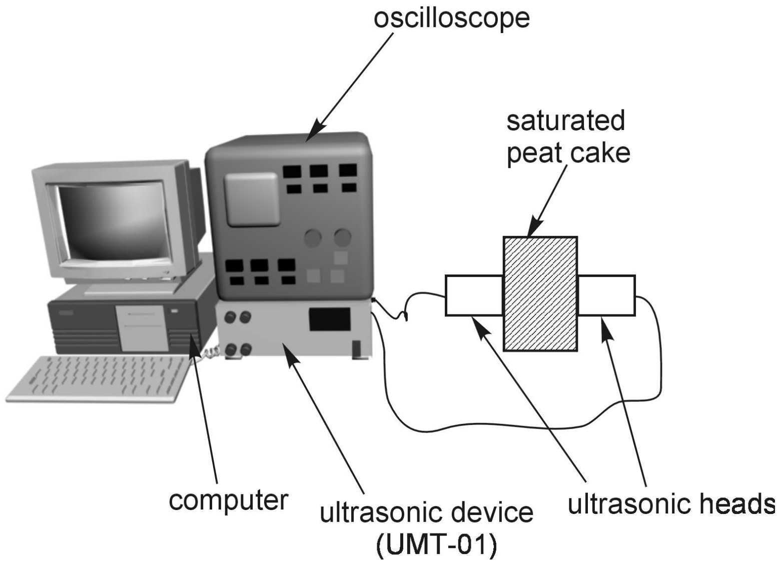

Fig. 7. Ultrasonic equipment.

Figure 8 presents the comparison of the ultrasonic wave velocity measured in the saturated peat cake (see Fig. 7) with the velocity of the fast wave. The reference velocity was estimated experimentally to be $c \approx 86$ m/s.

The theoretical curve is plotted for the following parameters: $a_{11} = 0.5$, $a_{22} = 0.4$, $a_{12} = 0.05$ and $\beta = 0.01$. Both the theoretical and experimental curves agree qualitatively very well.

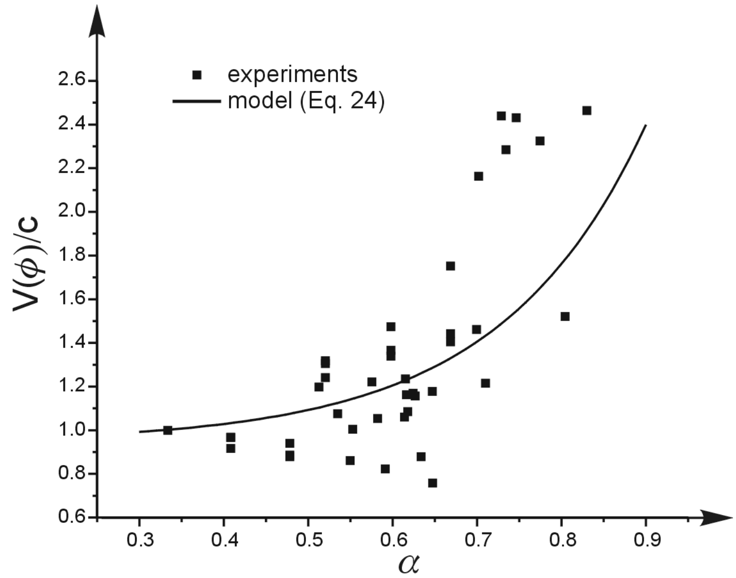

Fig. 8. Velocity of ultrasonic wave in peat compared with the phase velocity of the fast wave computed theoretically.

5. Final remarks

The model of wave propagation in sediments was developed for the purpose of analysis of the dispersion and attenuation properties of these media. The phase velocity dispersion curves and the attenuation coefficients for these waves were determined. The analysis presents us what can happen to the ultrasonic waves when they are passing through these media, and helps us to answer the question whether the ultrasounds can be used for identification of the mass fractions in sediments.

Our preliminary experiments show that it is possible to use the ultrasounds for determination of the solid mass fraction or the moisture content in sediments. Some difficulties which arose during our experimental studies, concerned mainly the proper choice of the power and frequency of the ultrasonic waves for the given suspension. If one chooses unsuitable frequency or power, the measurement of the density may prove to be not possible, because of dispersion and damping of the waves. The emitted wave may not reach the receiving head.

The theoretical models show us that the waves are attenuated and damped. It is shown that any ultrasonic wave generated in sediment may split into two parts, one of which is transported by the fast wave and the other by the slow one. The analysis shows that the slow waves are strongly damped, so the impulse transported by these waves may disappear on the way, and only the impulse transported by the fast wave has the chance to arrive at the receiving ultrasonic head. Figure 8 proves this because the high-frequency ultrasonic head of 600 kHz ($\beta = 0.05$) and not the low-frequency one of 40 kHz ($\beta = 0.75$) enabled us to make the measurements in the saturated peat cake.

As it results from Eqs. $(24)_2$ and $(25)_2$, the attenuation coefficient l_i depends on the coefficient b (related to fluid viscosity, porosity and permeability), the solid mass concentration α, elastic coefficients of the individual constituents a_{11}, a_{22}, a_{12} and the wave frequency ω. For the given medium, we can control only the wave frequency of ultrasonic waves, so it should be chosen properly to obtain the minimal attenuation coefficient.

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