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ON THE DIFFRACTION OF SOUND WAVE BY A WEDGE

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This paper presents calculations for plane wave diffraction by a right-angled wedge. Using the UFIMTSEV and OBERHETTINGER's theoretical approach, formulae are obtained for the diffracted field potential on the shaded wall of the wedge in the form of a series of cylindrical functions and a real integral. Some results of numerical calculations are also presented.

1. Introduction

More and more attention has been paid recently to the problems in the field of the applications of the theory of acoustic wave diffraction in the protection of the environment and of working posts. This field includes research related to all kinds of acoustic protecting devices, investigation of intensity decrease in rooms etc.

The present paper aims to discuss the following problem: to what extent one wall of a wedge (e.g. the corner of a building) is affected by a sound wave which propagates along the other wall (Fig. 1). The evident theoretical basis is here the theory of wave diffraction by a wedge of which the present problem is a special case.

Since it is impossible to find a compact solution of the problem of diffraction by a wedge, three theoretical approaches have been formulated to date:

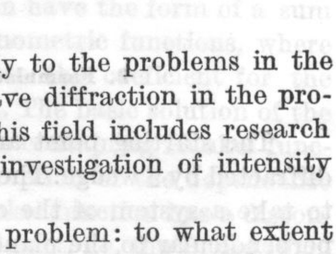

Fig. 1. A schematic diagram of the right-angled wedge

1) of SOMMERFELD [8, 9] which can be briefly summarized by saying that it uses the image source method and such a conformal transformation that straightens the wedge to a plane. This leads to an integral in a complex plane, which can be subsequently expanded into a series or calculated directly in a numerical way;

2) of OBERHETTINGER [4] which consists in relevant integral transformations of the function representing the incident wave and the diffracted wave, and subsequently in summing up of the two waves so that the boundary conditions on the wedge are satisfied. In turn, there follows an expansion into a series whose coefficients are found from these boundary conditions. An advantage of the OBERHETTINGER method is the interesting proposal of this author that an imaginary frequency should be formally introduced, thus simplifying the necessary mathematical operations and permitting a transition to pulses;

3) of UFIMTSEV [1] which proceeds in a direction different from those of the other two in that it assumes the acoustic potential in the form of a series and shows subsequently that it can be summed into a SOMMERFELD integral.

The initial part of the present paper is based on the UFIMTSEV theory, or rather part of it, which is adapted here to the present purposes and the mathematical part of which is developed later on.

2. Formulation of the problem and its analytical solution

The starting point are general formulae for the acoustic field of a wave diffracted by a wedge. The geometry of the wedge is shown in Fig. 2. It is possible to take a system of the cylindrical coordinates (r, φ, z) in which the axis z is perpendicular to the plane of the figure, the pole is placed at a point which is the trace of the point of the wedge on the plane of the figure and the angle φ is measured in a positive direction from the "upper" edge of the wedge. The source of the wave is a "thread" that is a straight line with densely set points radiating a cylindrical wave. Fig. 1 shows the trace of this straight line in the form of the point Q with the coordinates r_0 and φ_0. The desired field is sought at the point P with the coordinates r and φ. It is possible to begin with a formulation

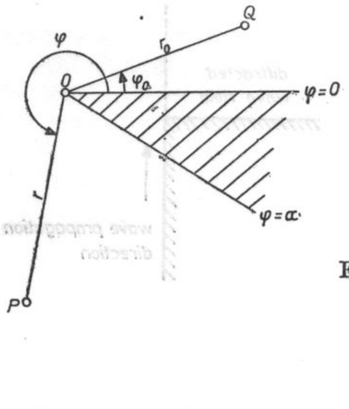

Fig. 2. Geometry of the problem of diffraction by a wedge in general case

of the general formulae and proceed subsequently to the limit $r_0 \to \infty$. This transition does not have to give, as UFIMTSEV proposed in [1], an exact value of the acoustic field potential for a plane wave, which can be certain only when diffraction by an object of finite dimensions is considered. In the case of an infinite edge it is possible to obtain a solution which can be used at relatively long distances from the edge of the wedge. It is interesting to add here that other works of the present authors in progress show that the point source field in space gives, in the case of the wedge, different expressions for the transition $r_0 \to \infty$ but both expressions agree for $r \to 0$.

Bearing in mind the fact that a cylindrical model of plane wave is used here, it is possible to proceed in the later part to the value $\alpha = \varphi = 3/2$. It is also possible to assume simultaneously the harmonic time dependence in the form $\exp(-\omega t)$. In the present case the boundary condition is the assumption that the walls of the wedge are perfectly rigid and therefore the acoustic pontential must satisfy the boundary conditions

$$\frac{\partial \Phi}{\partial \varphi} = 0 \quad \text{for } \varphi = 0 \text{ and } \varphi = \alpha. \tag{2.1}$$

It is known [2, 5, 6] that a solution of the Helmholtz equation for the acoustic potential in a cylindrical system of coordinates can have the form of a sum of terms, i.e. of the product of cylindrical and trigonometric functions, where the order of the cylindrical function must be equal to the coeficient for the angle φ in the argument of the trigonometric function. The basic solution of the Helmholtz equation should be the sum of components containing Hankel functions of the first kind (which is related to the assumption of the dependence $\exp(-i\omega t)$) and cosines. For $r = 0$, however, the Hankel function has a discontinuity of the type of $-\infty$ and therefore only the real part, i.e. the Bessel function, can be assumed. It is convenient to break the solution into two intervals: the Bessel function must occur for $r < r_0$ and the Hankel function for $r > r_0$. In order to make the solution continuous for $r = r_0$, the first solution must be multiplied by the Hankel function of r_0 and the other, by the Bessel function of r_0. This gives the solution in the form of the following series

$$\Phi(r, \varphi) = \begin{cases} \sum\limits_{s=0}^{\infty} c_s J_{r_s}(kr) H_{r_s}^{(1)}(k r_0) \cos r_s \varphi_0 \cos r_s \varphi, & r < r_0, \\ \sum\limits_{s=0}^{\infty} c_s J_{r_s}(kr_0) H_{r_s}^{(1)}(k r) \cos r_s \varphi_0 \cos r_s \varphi, & r > r_0, \end{cases} \tag{2.2}$$

where

$$r_s = s\frac{\pi}{\alpha}, \tag{2.3}$$

and J_{r_s} and $H_{r_s}^{(1)}$ denote, respectively, Bessel and Hankel functions of the first kind, of the order r_s. The choice of $H_{r_s}^{(1)}$ is, as was mentioned above, related to the choice of the time factor in the form $\exp(-i\omega t)$ [2, 6]. It can be seen from (2.2) and (2.3) that for $\varphi = 0$ and $\varphi = \alpha$ all the terms of the derivative $\partial\Phi/\partial\varphi$ become zero, thus satisfying the boundary condition (2.1).

The coefficients c_s can now be calculated. This can be done applying the identity

$$\int_s\int \frac{\partial\Phi}{\partial n} ds = \int_V\int\int \Delta\Phi dV \tag{2.4}$$

to a solid with its base limited by the contour L (Fig. 3) and the thickness dz. For $\Phi = \Phi(r, \varphi)$

$$\oint_L \frac{\partial\Phi}{\partial n} dl = \int_s \Delta\Phi d\sigma. \tag{2.5}$$

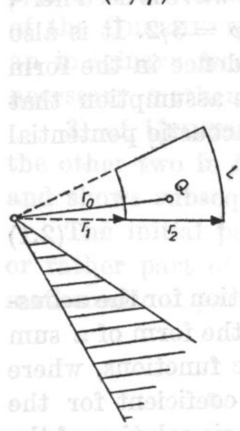

Fig. 3. Integration contour in formula (2.5)

Transition to the limit $r_1 \to r_0$ and $r_2 \to r_0$ gives

$$\int \left(\frac{\partial\Phi}{\partial r}\bigg|_{r_0+0} - \frac{\partial\Phi}{\partial r}\bigg|_{r_0-0} \right) r_0 d\varphi = \int_{s\to 0} \Delta\Phi d\sigma. \tag{2.6}$$

In the case of a linear source in space, and in the present case, of the point source Q, the acoustic potential must satisfy the inhomogeneous Helmholtz equation in the form [1, 5]

$$\Delta\Phi + k^2\Phi = A\frac{\delta(r-r_0, \varphi-\varphi_0)}{r}, \tag{2.7}$$

where A is a constant which can be normalized subsequently to the effectiveness of the source; $\delta(\ )$ is a Dirac distribution which when multiplied by r provides the integration properties of this distribution in a cylindrical coordinate system. In calculating $\Delta\Phi$ from equation (2.7), in order to substitute it on the right side of (2.6), it should be borne in mind that for $s \to 0$ integration of the term containing Φ gives a result tending to zero and that an integral containing the Dirac distribution δ only remains. Thus, from equation (2.6)

$$\int \left(\frac{\partial\Phi}{\partial r}\bigg|_{r_0+0} - \frac{\partial\Phi}{\partial r}\bigg|_{r_0-0} \right) r_0 d\varphi = A \lim_{r\to 0} \int_{s\to 0} \frac{\delta(r-r_0)\,\delta(\varphi-\varphi_0)}{r} r dr d\varphi. \tag{2.8}$$

Integration with respect to the variable r gives

$$\int \left(\frac{\partial \Phi}{\partial r} \bigg|_{r_0+0} - \frac{\partial \Phi}{\partial r} \bigg|_{r_0-0} \right) r_0 d\varphi = \frac{A}{r_0} \int \delta(\varphi - \varphi_0) r_0 d\varphi . \tag{2.9}$$

Since the size of the contour L is fully arbitrary, the identity of the integrals in formula (2.9) involves the equality of the integrands and thus

$$\frac{\partial \Phi}{\partial r} \bigg|_{r_0+0} - \frac{\partial \Phi}{\partial r} \bigg|_{r_0-0} = \frac{A}{r_0} \delta(\varphi - \varphi_0) . \tag{2.10}$$

It is possible to substitute on the left side of formula (2.10) the corresponding formula for Φ with $r < r_0$ and $r > r_0$, i.e. the first and second formulae of (2.2). Because of their complex form it is best to consider a single component of the sum first. Marking with a dash the integration with respect to the whole parameter under a cylindrical function, this gives

$$kc_s J_{r_s}(kr_0) H^{(1)\prime}_{r_s}(kr_0) - kc_s J'_{r_s}(kr_0) H^{(1)}_{r_s}(kr_0) =$$

$$= kc_s W \left[J_{r_s}(kr_0), H^{(1)}_{r_s}(kr_0) = kc_s \frac{2i}{\pi k r_0} \right], \tag{2.11}$$

where W is a wronskian which for the functions $J_{r_s}(kr_0)$ and $H^{(1)}_{r_s}(kr_0)$ has the form of (2.11) (cf. [7], p. 68). Formula (2.10) takes now the form

$$\frac{2i}{\pi r_0} \sum_{s=0}^{\infty} c_s \cos(r_s \varphi_0) \cos(r_s \varphi) = \frac{A}{r_0} \delta(\varphi - \varphi_0) . \tag{2.12}$$

Both sides of (2.12) can be divided by $r_0 \cos(r_s \varphi)$ and integrated with respect to φ in the interval from 0 to a. Since this interval must contain the value φ_0, the right side becomes $Ar_0^{-1} \cos(r_t \varphi_0)$. On the left side, however, because of the orthogonality of the system of the function $\cos(r_s \varphi)$, integration of the particular terms leads to their value of zero, except the term containing $s = t \neq 0$. Thus

$$\frac{2i}{\pi} c_t \cos(r \varphi_0) \int_0^a \cos^2(r_t \varphi) d\varphi = A \cos(r_t \varphi_0), \tag{2.13}$$

i.e.

$$c_t = \frac{A\pi}{ia} . \tag{2.14}$$

It can be seen that the coefficient c_s is constant for all $s \neq 0$. For $s = 0$, however, $\cos(r_s \varphi) = 1$, i.e. it can be noted easily that integration of the left side of (2.13) gives a result greater by a factor of two than before. It can thus be written jointly,

$$c_s = \varepsilon_s \frac{A\pi}{ia} , \tag{2.15}$$

where the symbol ε_s takes the values

$$\varepsilon_s = \begin{cases} \frac{1}{2}, & s = 0, \\ 1, & s \neq 0. \end{cases} \tag{2.16}$$

The formula for the potential becomes

$$\Phi(r, \varphi) = \begin{cases} \frac{A\pi}{ia} \sum_{s=0}^{\infty} \varepsilon_s J_{r_s}(kr) H_{r_s}^{(1)}(kr_0) \cos(r_s\varphi_0)\cos(r_s\varphi), & r < r_0, \\ \frac{A\pi}{ia} \sum_{s=0}^{\infty} \varepsilon_s J_{r_s}(kr_0) H_{r_s}^{(1)}(kr) \cos(r_s\varphi_0)\cos(r_s\varphi), & r > r_0. \end{cases} \tag{2.17}$$

Proceeding to the problem of a plane wave propagating along one of the walls of the wedge (with the qualification given above), the transition $r_0 \to \infty$ must first be considered. The following asymptotic formula can then be used for $H_{r_s}^{(1)}(kr_0)$ [2, 7],

$$H_{r_s}^{(1)}(kr_0) = \sqrt{\frac{2}{\pi k r_0}} \exp\left[i\left(kr_0 - \frac{\pi}{4} - \frac{\pi}{2} r_s\right)\right] = H_0^{(1)}(kr_0) \exp\left(-i\frac{\pi}{2} r_s\right). \tag{2.18}$$

What remains is only the acoustic field for $r < r_0$ and thus

$$\Phi(r, \varphi) = \frac{A\pi}{2ia} H_0^{(1)}(kr_0) \sum_{s=0}^{\infty} \varepsilon_s \exp\left(-i\frac{\pi}{2} r_s\right) J_{r_s}(kr)[\cos r_s(\varphi - \varphi_0) + \cos r_s(\varphi + \varphi_0)]. \tag{2.19}$$

In the case of a cylindrical wave source in a free space, the potential at a given point must be proportional to the function $H_0^{(1)}(kr_0)$ where r_0 is the distance of this point from the source. In the case when $kr_0 \gg l$ and $kr_0 \gg kr$ it can be assumed that for an arbitrary value of r the distance PQ (Fig. 2), which in reality is

$$PQ = \left(r^2 + r^2 - 2r\, r_0 \cos(\varphi - \varphi_0)\right)^{1/2} \tag{2.20}$$

can be taken for r_0. It can certainly be so assumed for a plane wave which corresponds to an infinitely great value of r_0. In general, the acoustic potential can be given by

$$\Phi(r, \varphi) = F[u(r, \varphi - \varphi_0) + u(r, \varphi + \varphi_0)], \tag{2.21}$$

where F is the amplitude of the free wave and the function u represents the diffraction phenomena.

Approximately, for large values of $kr_0 \gg kr$, for a cylindrical wave

$$F = \frac{A}{i} H_0^{(1)}(k\, r_0), \tag{2.22}$$

while for a plane wave from the direction of φ_0

$$\Phi_0 = A_0 \exp[-ikr\cos(\varphi-\varphi_0)] \tag{2.23}$$

and the acoustic potential of a diffracted wave can be represented in the form of the product of the amplitude A_0 and the function u, as in (2.21). In turn, the function defining the diffraction phenomena is equal to (when ψ denotes the value $\varphi-\varphi_0$ or $\varphi+\varphi_0$)

$$u(r,\psi) = \frac{\pi}{2\alpha}\sum_{s=0}^{\infty} \varepsilon_s \exp\left(-i\frac{\pi}{2}r_s\right) J_{r_s}(kr)\cos(r_s\psi). \tag{2.24}$$

It should be stressed that for $r\to 0$ $J_{r_s}(kr) = 0$, except for $s = 0$, since $J_0(0) = 1$. Therefore in the limits (for an arbitrary value of φ)

$$u(0,\psi) = \frac{\pi}{4\alpha}. \tag{2.25}$$

Formula (2.23) permits, if necessary, the constant A_0 to be normalized to the output of the source for a plane wave. This problem is not considered here in view of the aim of the present paper, i.e. a calculation of a decrease in the amplitude along the wall of the wedge. It is interesting to note, however, that it is useless to check here whether the solution assumed satisfies the so-called edge condition since UFIMTSEV himself reduces the results of his theory to a SOMMERFELD integral [8, 9] whose properties have been investigated in this respect.

3. The case of a plane wave propagating along the wall of the edge — the potential in the form of a series

Returning to the case of interest shown in Fig. 2 when the plane wave propagates along one of the walls of the wedge, and the interest here is in the acoustic field on the other wall, the following values occur in the formulae in section 2

$$\varphi_0 = 0, \qquad \varphi = \alpha = \frac{3\pi}{2}. \tag{3.1}$$

The index of the Bessel function under the sign of sum is now

$$r_s = s\frac{\pi}{\alpha} = \frac{2}{3}s. \tag{3.2}$$

Since for the values of the angles assumed

$$\varphi-\varphi_0 = \varphi+\varphi_0, \tag{3.3}$$

the difference $\varphi-\varphi_0$ or the sum $\varphi+\varphi_0$, symbolically represented by ψ, can in the present case be reduced to one value

$$\psi = \varphi = \alpha = \frac{3\pi}{2}, \tag{3.4}$$

i.e. the factor 2 occurs in the formula for Φ. Thus

$$\Phi(r) = \frac{2A_0}{3}\sum_{s=0}^{\infty}(-1)^s\varepsilon_s\exp\left(-i\frac{\pi}{3}s\right)J_{2s/3}(kr). \tag{3.5}$$

This gives $\Phi(r)$ in the form of a rapidly convergent series of Bessel functions.

Before making numerical calculations and drawing conclusions from the theory given here, it is useful to present a completely different approach to the same problem, which leads to an integral form of the expression for the potential Φ.

4. Integral expression for the potential

In order to reduce equation (2.24) to an integral form, it is possible to use the purely formal transition to an imaginary wave number, proposed by OBERHETTINGER [4], in the form

$$k = i\gamma, \tag{4.1}$$

and thus passing to the so-called modified Bessel functions. On the basis of the known relation for these functions ([3], 6.406),

$$I_\nu(z) = \exp\left(-\nu\frac{\pi}{2}i\right)J_\nu\left(z\exp\left(\frac{\pi}{2}i\right)\right), \tag{4.2}$$

it is possible to rewrite formula (2.24) in the form

$$u(r,\varphi) = \frac{\pi}{2\alpha}\sum_{s=0}^{\infty}\varepsilon_s I_{r_s}(\gamma r)\cos(r_s\psi). \tag{4.3}$$

In accordance with the aim of the present paper, it is possible to assume the boundary case $kr_0\to\infty$, i.e. formula (2.22) and that $\varphi_0 = 0$, and retaining still the arbitrary value of the angle, it is possible to write

$$\Phi(r,\varphi) = \frac{\pi}{\alpha}\sum_{s=0}^{\infty}\varepsilon_s I_{s\pi/\alpha}(\gamma r)\cos\left(\frac{s\pi}{\alpha}\varphi\right). \tag{4.4}$$

An integral representation of the modified Bessel function $I_p(z)$ ([3], 6.443) can now be used,

$$I_p(z) = \frac{1}{\pi}\int_0^\pi \exp(z\cos x)\cos(px)\,dx - \frac{\sin(p\pi)}{\pi}\int_0^\infty \exp(-z\,ch\,x)\exp(-px)\,dx. \quad (4.5)$$

Inserting expression (4.5) into (4.4), two modifications of this formula can be performed simultaneously: the order of summation and integration can be changed and summation can be performed up to an arbitrary finite N and then from N to infinity. This gives

$$a\Phi(r,\varphi) = \lim_{N\leftarrow\infty}\int_0^\pi \exp(\gamma r\cos x)\left\{\sum_{s=0}^{N}\varepsilon_s\cos\left(\frac{s\pi}{a}x\right)\cos\left(\frac{s\pi}{a}\varphi\right)\right\}dx +$$

$$-\int_0^\infty \exp(-\gamma r\,ch\,x)\left\{\sum_{s=0}^{\infty}\varepsilon_s\cos\left(\frac{s\pi}{a}\varphi\right)\sin\left(\frac{s\pi^2}{a}\right)\exp\left(-\frac{s\pi}{a}x\right)\right\}dx =$$

$$\stackrel{\text{df}}{=} \lim\int_0^\infty \exp(\gamma r\cos x)s_1(x,\varphi)\,dx - \int_0^\infty \exp(-\gamma r ch x)s_2(x,\varphi)\,dx. \quad (4.6)$$

The sums s_1 and s_2 in formula (4.6) can now be calculated. The first calculation, using elementary trigonometric formulae, gives the sum s_1 in the form

$$s_1 = \sum_{s=0}^{N}\varepsilon_s\left[\cos\frac{s\pi(x+\varphi)}{a} + \cos\frac{s\pi(x-\varphi)}{a}\right]. \quad (4.7)$$

The two sums on the right side of formula (4.7) can be gathered by means of a known formula ([3], 1.341.2), giving

$$s_1 = \frac{1}{2}\left\{\frac{\sin\left(\frac{\pi}{2a}(2N+1)(x+\varphi)\right)}{\sin\left(\frac{a}{2\pi}(x+\varphi)\right)} + \frac{\sin\left(\frac{\pi}{2a}(2N+1)(x-\varphi)\right)}{\sin\left(\frac{\pi}{2a}(x-\varphi)\right)}\right\}. \quad (4.8)$$

The components of the first expression tend in the limits for $N\to\infty$ to the Dirac distribution δ if the argument $x+\varphi$ and $x-\varphi$ [5]. It can be noted that when $\varphi > \pi$, the first of the integrals in (4.6) disappears, since the two values fall outside the integration interval.

In turn, the second sum in (4.6) can be transformed into the form

$$s_2 = \sum_{s=0}^{\infty}\varepsilon_s\exp\left(-s\frac{\pi}{a}x\right)\left[\sin\frac{s\pi(\pi+\varphi)}{a} + \sin\frac{s\pi(\pi-\varphi)}{a}\right] =$$

$$= \sum_{s=0}^{\infty}\exp\left(-s\frac{\pi}{a}x\right)\sin\frac{s\pi(\pi+\varphi)}{a} + \sum_{s=1}^{\infty}\exp\left(-s\frac{\pi}{a}x\right)\sin\frac{s\pi(\pi-\varphi)}{a}. \quad (4.9)$$

Using a relevant formula ([3], 1.461), it is possible to write that

$$s_2 = \frac{1}{2}\left(\frac{\sin\left(\frac{\pi}{\alpha}(\pi+\varphi)\right)}{ch\frac{\pi}{\alpha}x - \cos\left(\frac{\pi}{\alpha}(\pi+\varphi)\right)} + \frac{\sin\left(\frac{\pi}{\alpha}(\pi-\varphi)\right)}{ch\frac{\pi}{\alpha}x - \cos\left(\frac{\pi}{\alpha}(\pi-\varphi)\right)}\right). \tag{4.10}$$

The form of the sum can now be determined for the case of interest when $\varphi = \alpha = \frac{3}{2}\pi$. Substitution and calculation of the values of the relevant trigonometric functions give

$$s_2 = \frac{\sqrt{3}}{2}\frac{1}{ch\frac{2}{3}x - \frac{1}{2}}. \tag{4.11}$$

Substitution of this result into (4.6) and consideration that the first integral disappears lead to

$$\Phi\left(r, \frac{3}{2}\pi\right) = \frac{1}{\sqrt{3}\pi}\int_0^\infty \frac{\exp(-\gamma r ch x)}{ch\frac{2}{3}x - \frac{1}{2}}dx. \tag{4.12}$$

It is necessary to return now to the real value of the wave number k, assuming in formula (4.12) that $\gamma = -ik$ (cf. (4.1)). This gives the final formula which expresses in integral form the acoustic potential along the wall of the edge

$$\Phi\left(r, \frac{3}{2}\pi\right) = \frac{1}{\sqrt{3}\pi}\int_0^\infty \frac{\exp(ikr\, ch\, x)}{ch\frac{2}{3}x - \frac{1}{2}}dx. \tag{4.13}$$

The integral obtained on the real semi-axis is a rapidly converging one and, in addition to (3.5), can be used to calculate the diffracted field of a plane wave on the wall of the wedge.

5. Conclusions

On the basis of the final formula (4.13), which gives in integral form the expression for the acoustic potential of a plane wave diffracted by a right-angled wedge, numerical calculations were made of the squared value of the modulus of the sound pressure on the wall of the wedge. The following formula which is valid in the case of a harmonic time dependence was used

$$|p| = \varrho\omega|\Phi|. \tag{5.1}$$

The results of the numerical calculations are given in Figs. 4 and 5. Fig. 4 shows, in dB, a decrease in the sound pressure on the wedge as a function of the relative distance $kr = 2\pi r/\lambda$ (lower curve) and, for comparison, for a spherical wave (dashed curve). Since the source of a spherical wave, placed on the edge of the wedge, would have to show there an infinite value of the sound pressure, therefore in this case its value at a point where $kr = l$ was assumed

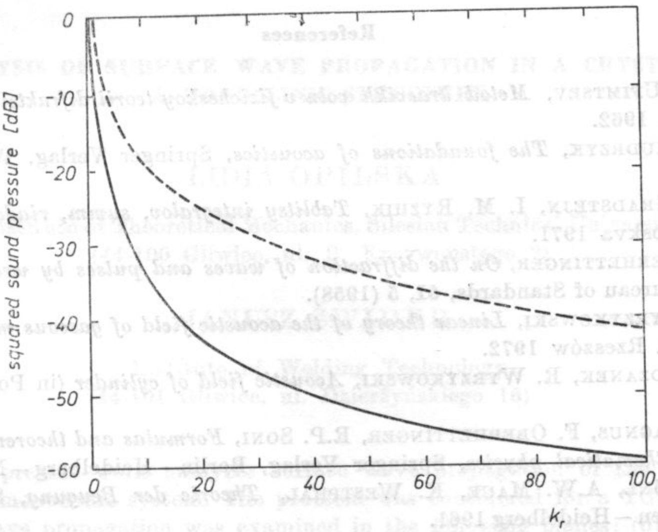

Fig. 4. Drop of the level of the squared modulus of sound pressure along the shaded wall of the right-angled wedge as a function of the normalized distance from the edge (solid line), compared with the curve characteristic of a spherical wave (dashed line)

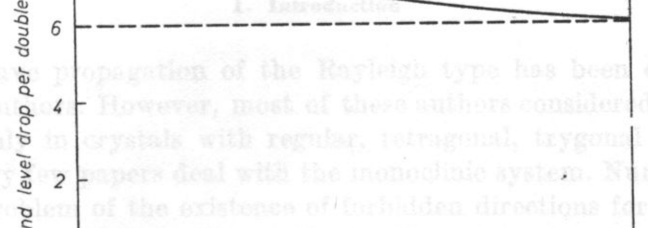

Fig. 5. Drop in the level of the squared modulus of sound pressure along the shaded wall of the right-angled wedge per double distance (solid line), compared with the value characteristic of a spherical wave (dashed line)

as the reference point. Fig. 5 shows a drop in the sound pressure per double distance as a function of a double distance.

It can be seen in the two diagrams that the sound pressure on the wedge decreases initially at a much faster rate than that for a spherical wave and subsequently stabilizes on the level characteristic for a spherical wave.

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