ARCHIVES OF ACOUSTICS
14, 1-2, 3-10 (1989)

LIGHT AND ACOUSTIC PULSE INTERACTION IN THE BRAGG DIFFRACTION REGION

JERZY BODZENTA, ZYGMUNT KLESZCZEWSKI

Institute of Physics, Technical University of Silesia
(44–100 Gliwice, ul. Krzywoustego 2)

This paper presents results of a theoretical analysis of interactions between a monochromatic light wave and series of acoustic pulses in the Bragg diffraction region. An expression for the electric field intensity of a diffracted light wave was achieved on the basis of known theories. The intensity is a sum of harmonics formed due to the diffraction of inciding light onto harmonics of the acoustic wave. Individual components of the light wave resulting from diffraction are spatially separated. A numeric analysis performed for a series of perfect, rectangular acoustic pulses proved that the measurement of the angular distribution of light intensity in a diffracted beam leads to the analysis of the acoustic wave's amplitudes spectrum. Therefore, an acoustic wave spectrum analyzer can be built with the utilization of the described interaction. The possibility of measuring parameters of acoustic pulses directly during propagation in the medium without influencing these parameters is the main advantage of such an analyzer. The frequency band of the proposed analyzer is limited by the Bragg condition depends on acoustic and acousto-optics properties of the medium and the geometry of the interaction region.

W pracy przedstawiono wyniki teoretycznej analizy oddziaływania monochromatycznej fali świetlnej z ciągiem impulsów akustycznych w obszarze dyfrakcji Bragga. Na podstawie znanych teorii otrzymano wyrażenie opisujące natężenie pola elektrycznego ugiętej fali świetlnej. Natężenie to jest sumą składowych harmonicznych powstających w wyniku dyfrakcji padającego światła na składowych harmonicznych fali akustycznej. Poszczególne składowe fali świetlnej powstałej w rezultacie dyfrakcji są rozdzielone przestrzennie. Na podstawie analizy numerycznej przeprowadzonej dla ciągu idealnych, prostokątnych impulsów akustycznych wykazano, że pomiar rozkładu kątowego natężenia światła w wiązce ugiętej pozwala na analizę widma amplitud fali akustycznej. Tak więc opisywane oddziaływanie stwarza możliwość budowy analizatora widma fali akustycznej. Do głównych zalet takiego analizatora należy zaliczyć możliwość pomiaru parametrów impulsów akustycznych bezpośrednio w czasie ich propagacji w ośrodku bez zmiany tych parametrów w czasie pomiaru. Pasmo przenoszenia proponowanego analizatora jest ograniczone koniecznością wypełnienia warunku Bragga i zależy od własności akustycznych i akustooptycznych ośrodka i geometrii obszaru oddziaływania.

1. Introduction

The light-acoustic pulses interaction is one of the more important problems in acousto-optics arousing interest during the last several years. However hitherto published papers are mainly concerned with the Raman–Nath type of diffraction [1–3, 5–7].

This paper presents a theoretical analysis of light diffraction on acoustic pulses in the Bragg region.

On the basis of known theories of light diffraction on a continuous acoustic wave an angular distribution of diffracted light wave component intensities resulting from light-acoustic wave harmonics interaction. Also results of numeric calculations are presented.

2. Theoretical analysis

Let us assume an acoustic wave in the form of rectangular pulses with duration τ, repeating period T and carrier frequency Ω_0. A series of such pulses is shown in Fig. 1.

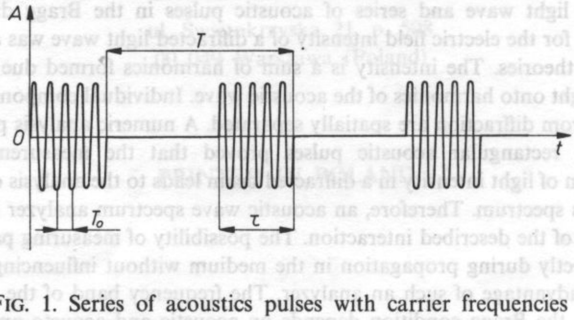

FIG. 1. Series of acoustics pulses with carrier frequencies Ω_0

Such a wave can be described with function $\Phi(\bar{r}, t)$ which is a sum of harmonics

$$\Phi(\bar{r}, t) = \sum_{l=-n}^{n} A_l \exp\{i[(\Omega_0 + l \cdot \Delta\Omega) \cdot t - \bar{K}_l \bar{r}]\} \tag{1}$$

where A_l amplitude of the l-component, $\Delta\Omega = \dfrac{2\pi}{T}$; $\Omega_0 = \dfrac{2\pi}{T_0}$, K_l wave vector of the wave l-component, equal to $\dfrac{\Omega_0 + l \cdot \Delta\Omega}{v}$, v propagation velocity of the acoustic wave, $\bar{r}$ vector of position.

Let us accept the geometry of interaction presented in Fig. 2. The acoustic wave propagates along the z-axis. L is the width of the acoustic beam. The light wave

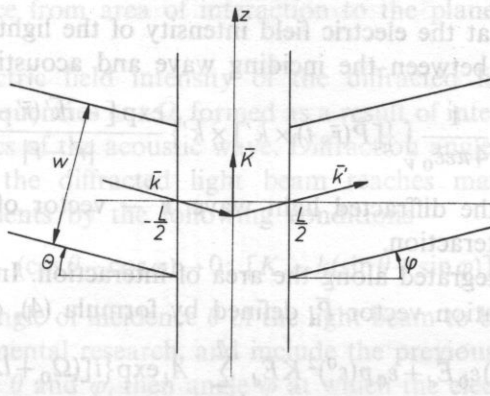

FIG. 2. Geometry of the interaction area

propagates in the *xz* plane under angle θ to the *x*-axis. The intersection of the light beam forms a square with side equal to *w*.

Let us assume also that these waves have constant intensity in the area of interaction. The diffracted light wave propagates in the *xz* plane under angle φ to the *x*-axis.

The electric field intensity of an inciding monochromatic light wave can be noted as

$$\bar{E}_p = \bar{E}_0 \exp[i(\omega t - \bar{k}\bar{r})], \tag{2}$$

where ω, $\bar{k}$ — frequency and wave vector respectively of the light wave.

It can be accepted that the diffracted light wave is formed as an interference result of secondary electromagnetic waves emitted by electric dipole moments, induced by the inciding light wave. By changing permittivity in the area of interaction acoustic wave causes changes of the electric polarization vector. If we accept the medium as an isotropic, non-magneti dielectric, then the vector of polarization is equal to

$$\bar{P} = (\varepsilon_0 - 1)\varepsilon_0 \bar{E}_p, \tag{3}$$

where: ε_0 — permittivity of free space, ε — relative permittivity of the medium.

When an acoustic wave propagates in the medium then the permittivity of the medium is a sum of two components: constant component ε^0 — corresponding with permittivity without disturbance, and modulating term related to the disturbance of the medium:

$$\varepsilon = \varepsilon^0 + \Delta\varepsilon = \varepsilon^0 - (\varepsilon^0)^2 ps, \tag{4}$$

where p — effective value of the medium's photo-elastic constant, s — deformation of the medium due to transition of acoustic wave.

It can be proved that the electric field intensity of the light wave generated as a result of interaction between the inciding wave and acoustic wave is equal to

$$\bar{E}_d(\bar{r};\, t) = \frac{1}{4\pi\varepsilon\varepsilon_0} \int_V \{[\bar{P}(\bar{r},\, t)\times \bar{k}']\times \bar{k}'\} \frac{\exp[-ik'(\bar{r}'-\bar{r})]}{|\bar{r}'-\bar{r}|}\, dV, \tag{5}$$

where $\bar{k}$ — vector of the diffracted light wave, $\bar{r}$ — vector of position of points outside the area of interaction.

Expression (5) is integrated along the area of interaction. Including expressions (1) and (4) the polarization vector $\bar{P}$, defined by formula (4), can be expressed as

$$\bar{P}(\bar{r}, t) = (\varepsilon^0-1)\varepsilon_0 \bar{E}_d + \varepsilon_0 p(\varepsilon^0)^2 K\bar{E}_d \sum_{l=-n}^{n} A_l \exp\{i[(\Omega_0 + l\Delta\Omega)t - \bar{K}_l\bar{r}]\}, \tag{6}$$

where $s = -K\Phi$, K — mean value of the wave number for acoustic wave.

The first term in expression (6) will be disregarded in further considerations, because it does not influence the generated of the diffracted light wave. Substituting (6) in formula (3) we reach

$$\bar{E}_d = \frac{1}{4\pi\varepsilon_0} \sum_{l=-n}^{n} \int_V \{[\bar{P}_{0l}\times \bar{k}']\times \bar{k}'\} \exp[-i(\bar{k}+\bar{K}_l)\bar{r}] \frac{\exp[-ik'|\bar{r}'-\bar{r}|]}{|\bar{r}'-\bar{r}|} dV, \tag{7}$$

where

$$\bar{P}_{0l} = \varepsilon_0 \varepsilon p K A_l \exp[i(\omega+\Omega_l)t]\cdot \bar{E}^0, \tag{8}$$

$\bar{E}^0$ — amplitude of electric field intensity of inciding light wave

$$\Omega_l = \Omega_0 + l\cdot \Delta\Omega$$

and it was also accepted that $\varepsilon^0 \cong \varepsilon$. We accept that $\bar{E}^0 \perp \bar{k}'$ in order to simplify further considerations. Then

$$\{[\bar{P}_{0l}\times \bar{k}']\times \bar{k}'\} = -k^2 P_{0l}. \tag{9}$$

As a result the expression (7) in scalar notation will have the following form

$$E_d = \frac{k^2}{4\pi\varepsilon_0} \sum_{l=-n}^{n} \int_V P_{0l} \exp[-i(\bar{k}+\bar{K}_l)\bar{r}] \frac{\exp[-ik'|\bar{r}'-\bar{r}|]}{|\bar{r}'-\bar{r}|} dV. \tag{10}$$

The diffraction pattern is most frequently observed at a much greater distance in relation to linear dimensions of the diffraction area. Furthermore we can accept that $k \cong k'$ because $k \gg K$. This also means that angles θ and φ are small angles.

Calculating the integral in expression (10), with mentioned above assumptions, we reach the final expression for E_d

$$E_d = -\frac{k^2 K\varepsilon p}{4\pi} E_0 \frac{\exp(-ikd\cdot\cos\varphi)}{d\cdot\cos\varphi} \frac{W^2 L}{\cos\theta} \frac{\sin[k(\cos\theta-\cos\varphi)L/2]}{k(\cos\theta-\cos\varphi)L/2} \times$$

$$\times \sum_{l=-n}^{n} A_l \frac{\sin\{[K_l - k(\sin\theta+\sin\varphi)]w/2}{[K_l - k(\sin\theta+\sin\varphi)]w/2} \exp[i(\omega+\Omega_l)t], \tag{11}$$

where d – distance from area of interaction to the plane in which diffraction is observed.

Therefore, the electric field intensity of the diffracted light wave is a sum of harmonics with frequencies $\omega+\Omega_l$ formed as a result of interaction between inciding light and harmonics of the acoustic wave. Diffraction angles φ at which the electric field intensity of the diffracted light beam reaches maximum are defined for individual components by the following conditions

$$(\cos\theta-\cos\varphi)\rightarrow 0;\ [K_l-k(\sin\theta+\sin\varphi)]\rightarrow 0 \tag{12}$$

If we assume the angle of incidence θ of the light beam to be constant, what usually happens in experimental research, and include the previous assumption concerning the value of angles θ and φ, then angle φ at which the electric field intensity of the l-component of the diffracted wave reaches maximum is expressed

$$K_l-k(\sin\varphi+\sin\theta)=0. \tag{13}$$

This means that individual harmonics of the diffracted light wave will be distributed in space and will be relatively simple to measure. The following formula expresses light intensities of individual components

$$I_{dl}=\sqrt{\frac{\varepsilon_0}{\mu_0}}\left|E_{dl}\right|^2, \tag{14}$$

where

$$E_{dl}=\frac{k^2K\varepsilon pE^0}{4\pi}\frac{\exp[-jkd\cos\varphi]}{d\cdot\cos\varphi}\frac{w^2L}{\cos\theta}\frac{\sin\left[k(\cos\theta-\cos\varphi)\frac{L}{2}\right]}{\left[k(\cos\theta-\cos\varphi)\frac{L}{2}\right]}\times$$

$$\times A_l\frac{\sin\left\{[K_l-k(\sin\theta+\sin\varphi)]\frac{w}{2}\right\}}{[K_l-k(\sin\theta+\sin\varphi)]\frac{w}{2}}\exp[j(\omega+\Omega_l)\cdot t]$$

After substitution we have

$$I_{dl}=\frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\left\{\frac{k^2K\varepsilon pE^0}{4\pi}\frac{w^2L}{d\cdot\cos\varphi\cos\theta}\frac{\sin\left[k(\cos\theta-\cos\varphi)\frac{L}{2}\right]}{\left[k(\cos\theta-\cos\varphi)\frac{L}{2}\right]}\times\right.$$

$$\left.\times\frac{\sin\left\{[K_l-k(\sin\theta+\sin\varphi)]\frac{w}{2}\right\}}{[K_l-k(\sin\theta+\sin\varphi)]\frac{w}{2}}\right\}^2|A_l|^2 \tag{15}$$

3. Numerical calculations

The angular distribution of component intensities of a light wave diffracted on a series of ideal rectangular pulses shown in Fig. 1 was calculated on the basis of expression (15). In the case under consideration amplitudes of individual components of the acoustic wave are

$$A_l = \frac{\tau \cdot \Delta\Omega}{2\pi} \frac{\sin\left(l \cdot \Delta\Omega \frac{\tau}{2}\right)}{l \cdot \Delta\Omega\tau/2}. \tag{16}$$

If a light wave incides onto the area of interaction under the angle $\theta = \arcsin(K_0/2k)$, equal to the Bragg angle for the carrier frequency of the acoustic wave and defined by condition (13), then the ratio of the intensity of the l-component to the maximal intensity of the central component $l = 0$ is as follows

$$\frac{I_{dl}}{I_{d0}} = \left(\frac{\cos\theta}{\cos\varphi}\right)^2 \left\{ \frac{\sin(l \cdot \Delta\Omega \cdot \tau/2)}{l \cdot \Delta\Omega \cdot \tau/2} \frac{\sin\left[k(\cos\theta - \cos\varphi)\frac{L}{2}\right]}{k(\cos\theta - \cos\varphi)\frac{L}{2}} \times \right.$$

$$\left. \times \frac{\sin\left\{[K_l - k(\sin\theta + \sin\varphi)]\frac{w}{2}\right\}}{[K_l - k(\sin\theta + \sin\varphi)]\frac{w}{2}} \right\}^2. \tag{17}$$

The dependence of the l-component's intensity upon the φ angle is determined by the square of the product of $\sin x/x$ type functions; while for small θ and φ angles the last term of the product, in braces, changes most quickly with a change of the φ angle. This term conditions the width of the diffraction maximum of given component. Fig. 3 presents maximal values of (I_{dl}/I_{d0}) for components of a diffracted light wave as a function of the number of component l. Two additional axes have been drawn in the diagram. Diffraction angles φ corresponding with individual components and frequencies of acoustic wave's components Ω_l for which diffraction occurs are marked on them. Calculations were performed for following values of parameters found in (17):

$$\frac{\Omega_0}{2\pi} = 500\,\text{MHz}; \quad \frac{\Delta\Omega}{2\pi} = 1\,\text{MHz}; \quad \tau = 10^{-7}\,\text{s}; \quad L = 2 \cdot 10^{-3}\,\text{m};$$

$$w = 10^{-2}\,\text{m}; \quad \lambda = \frac{2\pi}{k} = 630\,\text{nm}; \quad v = 5000\frac{\text{m}}{\text{s}}.$$

Angle of incidence, calculated from the Bragg condition $\theta = 1°48'18''$.

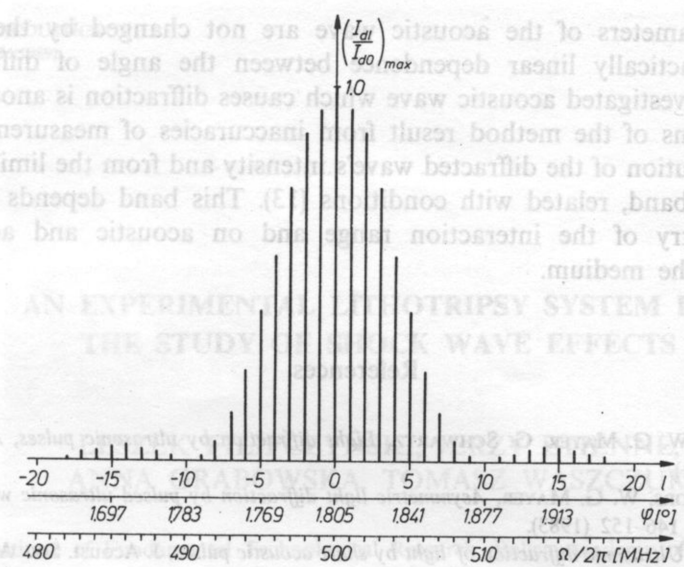

FIG. 3. Dependence of maximal values of relative intensities of diffracted light waves harmonics of the number of component l, angle of diffraction φ and frequency of component of acoustic wave Ω_l, which causes diffraction

Calculations were carried out in the range of angles expressed by inequality

$$1°36'18'' \leqslant \varphi \leqslant 2°0'18''$$

with step $\Delta\varphi = 10''$.

4. Conclusion

It results from our considerations that the interaction of a light wave with acoustic pulses provides a possibility of analysing harmonics of these pulses. Equation (15) and Fig. 3 show that the measurement of the angular distribution of light intensity in a diffracted wave can lead to the determination of the amplitude spectrum a series of acoustic pulses, calculated in the Fourier transform. Owing to the spatial resolution of individual components of the diffracted light wave measurements do not require complicated measuring methods. The direct measurement of acoustic pulses propagating in a given medium, without the necessity of converting them into electric signals (what inevitabely leads to deformations) is one of the main advantages of the spectral analysis of acoustic pulses based on the described interaction. It is also important that acousto-optic interaction do not influence the propagation of an acoustic wave when non-linear effects do not occur.

Therefore, parameters of the acoustic wave are not changed by the measuring process. A practically linear dependence between the angle of diffraction and frequency of investigated acoustic wave which causes diffraction is another facilitation. Limitations of the method result from inaccuracies of measurements of the angular distribution of the diffracted wave's intensity and from the limited width of the frequency band, related with conditions (13). This band depends also on the chosen geometry of the interaction range and on acoustic and acousto-optic properties of the medium.

References

[1] E. HÄUSLER, W. G. MAYER, G. SCHWARTZ, *Light diffraction by ultrasonic pulses*, Acoust. Lett. **4**, 180–194 (1981).

[2] T. H. NEIGHBORS, W. G. MAYER, *Asymmetric light diffraction by pulsed ultrasonic waves*, J. Acoust. Soc. Am. **74**, 146–152 (1983).

[3] R. N. ZITTER, *Ultrasonic diffraction of light by short acoustic pulses*, J. Acoust. Soc. Am., **43**, 864–870 (1968).

[4] R. W. DAMON, W. T. MALONEY, D. H. MC MAHON in: *Физическая акустика*, [Ed.] W. P. Mason, R. N. Thurston, tom 7, Mir, Moskwa 1974, 327–336.

[5] L. JAKAB, P. RICHTER, *Spectrum analyzer with acousto-optic Bragg cell*, Proc. 3rd Spring School on Acousto-optics and Applications, Wieżyca, Gdańsk 1986, 106–112.

[6] E. MARKIEWICZ, *Some numerical results for light diffraction on ultrasonic pulses*, Proc. 3rd Spring School on Acousto-optics and Applications, Wieżyca, Gdańsk 1986, 34–45.

[7] W. G. MAYER, T. H. NEIGHBORS, *Acusto-optic interactions by pulsed ultrasound*, Proc. 3rd Spring School on Acousto-optics and Applications, Wieżyca, Gdańsk 1986, 8–21.

Received on June 16, 1987, revised version on April 6, 1988.