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RADIATION OF PIEZOELECTRIC RINGS INTO THE AIR THROUGH CYLINDRICAL WAVEGUIDES

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In the paper the results of the investigations of the radiation of piezoelectric rings into the air through solid (e.g. glass, plexiglass, aluminium) cylinders are presented. The obtained radiation patterns approximate Bessel distribution. In the case of short cylinders the vibrations are more complicated, one obtains zero-order Bessel function with several zero-crossings. For long cylinders Bessel function is truncated, the quantity of sidelobes is reduced. The obtained results show that the configurations: piezoelectric ring–cylinder can be applied as ultrasonic transducers with narrow beam with limited diffraction in certain range. The additional advantage of this construction is the increase of vibration amplitude on the transducer axis, the disadvantage — small frequency bandwidth.

1. Introduction

Ultrasonic transducers with limited diffraction beams are an improvement in the field of object and environment recognition, in nondestructive testing and in medical imaging. They are competitive in relation to the focusing transducers. Complicated focus shift is necessary during the work of focusing transducers because the energy concentration occurs in the definite distance from the transducer. Increased depth of field can be obtained using arrays with dynamically focused transmission [11] or axicon transducers [3]. An acoustic beam with limited diffraction can be obtained using a transducer with Bessel distribution of vibrations on its surface. However such a beam has limited range and high sidelobes.

Large literature exists (e.g. [4, 11, 16]) concerning transducers with limited diffraction beams (Bessel beams or X waves) but still their theory is not complet. Durnin's theory [5] concerns in principle unlimited Bessel distribution of vibration. Its application to the ultrasonic waves requires to introduce truncated Bessel functions because apertures used in practice are finite. The important problem is also the way of sidelobes reduction [11]. It is necessary to analyze more precisely the near and far field and the influence of distribution disturbances in the form of additional functions.

The application of more general methods of calculations of radiation characteristics for transducers with nonuniform vibration distribution on the surface leads to the statement of differences in shapes of characteristics. These differences are caused by deviations from ideal Bessel distribution. To this end the transfer function method, developed by the authors, is very useful [8, 9].

The aim of this work is to apply the natural vibration distributions, which approximate Bessel distribution, to transducer constructions. It is known that the deformations occurring during the resonant vibrations of rings, membranes and cylinders can be described by combinations of Bessel functions. On the ground of the analysis of the theoretical works connecting the vibrations of rings and cylinders, e.g. [1, 13, 14], one can presume that certain modes of these vibrations can be used as a source of acoustic waves.

2. Ring radiation to a solid medium

2.1. Narrow ring ($R \gg \lambda$, $a_2 - a_1 < \lambda$)

A narrow ring with uniform amplitude of vibration in the axis direction gives an acoustic field with the pressure distribution described by zero-order Bessel function:

$$\frac{p}{p_0} = J_0\left(\frac{2\pi R}{\lambda}\sin\theta\right), \tag{2.1}$$

where: R — ring radius, λ — wavelength in the acoustic medium, θ — radiation directivity angle.
To satisfy the condition $R \gg \lambda$ for a solid medium is more difficult than for a gaseous medium. It requires to use the ring with large diameter or high frequency. Therefore the application of a ring to the generation of acoustic field with Bessel distribution is advantageous in megahertz frequency range. In this case the interference near field occurs in the form of two lobes, which in the certain distance transform in one central lobe and several sidelobes, in accordance with above mentioned formula.

2.2. Wide ring

For a wide ring the situation is more complicated. The uniform distribution of vibration amplitude on the surface leads to a complicated structure of acoustic field. This structure is more complicated than for the distributions described by Gauss function or truncated Bessel function.

In the literature one can find the approximation formulae for the calculation of acoustic field of a wide ring with uniform distribution of vibration amplitude. These formulae contain higher order Bessel functions [2]:

$$\frac{p}{p_0}=(a_2^2-a_1^2)^{-1}\left| a_2^1\frac{2J_1(x_2)}{x_2}-a_1^2\frac{2J_1(x_1)}{x_1}\right| \tag{2.2}$$

where: a_1 — inner radius of a ring, a_2 — outer radius of a ring, $x_1 = a_1 k\sin\theta$, $x_2=a_2 k\sin\theta$, $k=2\pi/\lambda$.
Besides the analytic formula one can use the transfer function method [8] for calculations of acoustic fields with optional parameters.

Figure 1 presents the acoustic field profile in the polystyrene at the distance 5 cm from the ring, calculated using transfer function method and the formula (2.2). The difference is not large, transfer function method gives more exact results. As can be seen the acoustic field distribution at this distance corresponds to Bessel function.

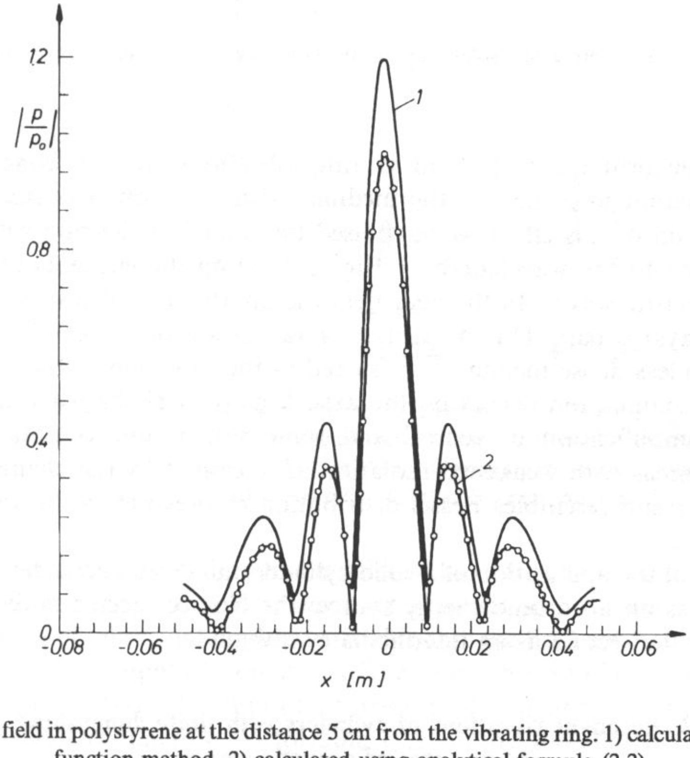


Fig. 1. Acoustic field in polystyrene at the distance 5 cm from the vibrating ring. 1) calculated using transfer function method, 2) calculated using analytical formula (2.2).

The application of piezoceramic ring for acoustic wave generation in gaseous media is not efficient owing to strong mismatching of the acoustic impedances. One can reduce impedance mismatching and increase ultrasonic energy transmission into the air using transducers consisting of one or some rings of piezoelectric ceramic embedded in plastic material [10]. The other method is to use an intermediate element in the form of a cylinder made of e.g. glass, plexiglass, aluminium. Such a cylinder

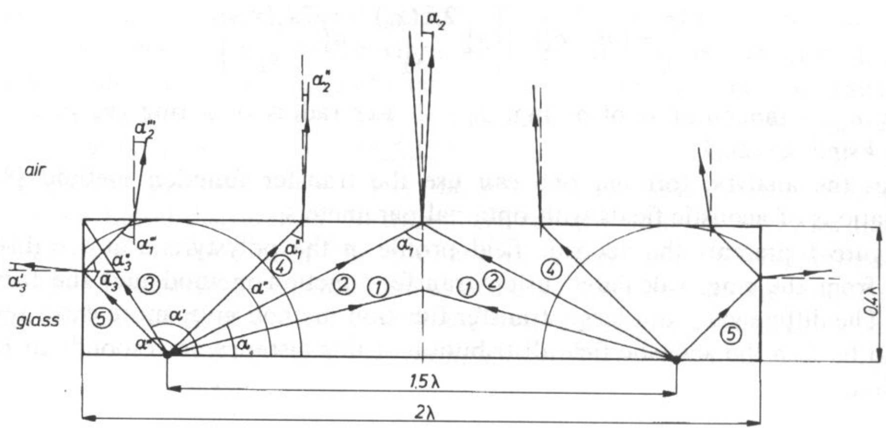


Fig. 2. Influence of an intermediate solid plate on the convergence of acoustic wave beam in the air.

permits to transform the amplitude of ring vibrations. It also concentrates the ultrasonic radiation into the air, the medium with very low acoustic impedance. For the depiction of this effect we have used the model of the ring with the width small in relation to the wavelength — Fig. 2, treating the elements of the ring as a source of acoustic waves. In the near field the interference of waves (presented in the form of rays) occurs. On the surface a ray going out from the more dense medium to the less dense medium is refracted in the direction of the configuration axis. The rays summation occurs on the axis. Finally after the wave transit of the cylinder the amplification of central and some side beams is obtained. Simultaneously the areas with weakened radiation arise caused by the change of the ray phase. Such a result resembles Bessel distribution of pressure in the vicinity of the cylinder surface.

In the case of the application of a solid cylinder one should consider two variants of its activity as an ultrasonic energy source: the first connected with its resonant vibrations and the second as an intermediate waveguide.

3. Resonant vibrations of cylinders with finite dimensions as a source of acoustic waves

The question of vibrations of cylinders with finite dimensions was the subject of many theoretical [6, 7, 15] and experimental [12, 17] works. However the problem has not been solved analytically in the general case because of the complicated boundary conditions. There are the solutions of the particular cases — the long rod or the thin plate. The other cases lead to the complicated equations which can be solved numerically. In these cases many modes exist. Only some modes have practical value and can be applied for ultrasonic energy radiation.

4. Vibrations of configurations: piezoelectric ring—cylinder

In ultrasonic transducers a metal cylinder is used as a constituent element with a piezoelectric plate or ring. The piezoelectric element excites resonant vibrations with large amplitude, which are the source of radiation. One could suppose that the strongest ultrasonic effects can be obtained in the case of resonant vibrations of the cylinder. However, as experiments indicate, the strongest ultrasonic effects are obtained in the case of resonant vibrations connected with the piezoelectric element — Fig. 3. In this case the cylinder is a waveguide which transmits the vibrations of the

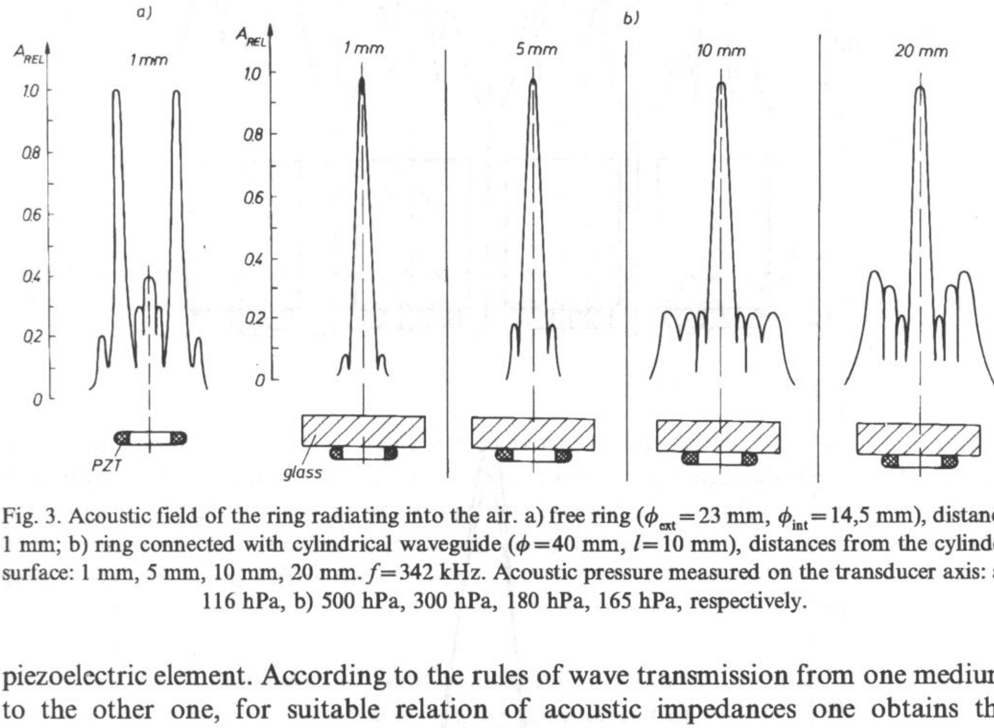


Fig. 3. Acoustic field of the ring radiating into the air. a) free ring ($\phi_{ext}=23$ mm, $\phi_{int}=14,5$ mm), distance 1 mm; b) ring connected with cylindrical waveguide ($\phi=40$ mm, $l=10$ mm), distances from the cylinder surface: 1 mm, 5 mm, 10 mm, 20 mm. $f=342$ kHz. Acoustic pressure measured on the transducer axis: a) 116 hPa, b) 500 hPa, 300 hPa, 180 hPa, 165 hPa, respectively.

piezoelectric element. According to the rules of wave transmission from one medium to the other one, for suitable relation of acoustic impedances one obtains the transformation of vibration amplitude. In this case the field generated in the cylinder and the vibration amplitude on its surface play an essential part. According to formula (2.1) or (2.2) the distribution of stress inside a cylinder with large diameter is described by Bessel function — formula (2.1) or by the sum of Bessel functions — formula (2.2) for a wide ring. The distribution of vibration amplitude on the cylinder surface corresponds to this stress distribution. The vibrations on the cylinder surface are the source of ultrasound in a gaseous medium. The pressure measured in the vicinity of vibrating cylinder surface has the profile which approximates the distribution described by Bessel function — Fig. 4. In this way it is possible to realize the transducer with Bessel distribution of vibration amplitude using a piezoelectric ring and a solid cylinder. The vibrations with such amplitude distribution can be applied for generation of nondiffracting acoustic beam in the air. Using different

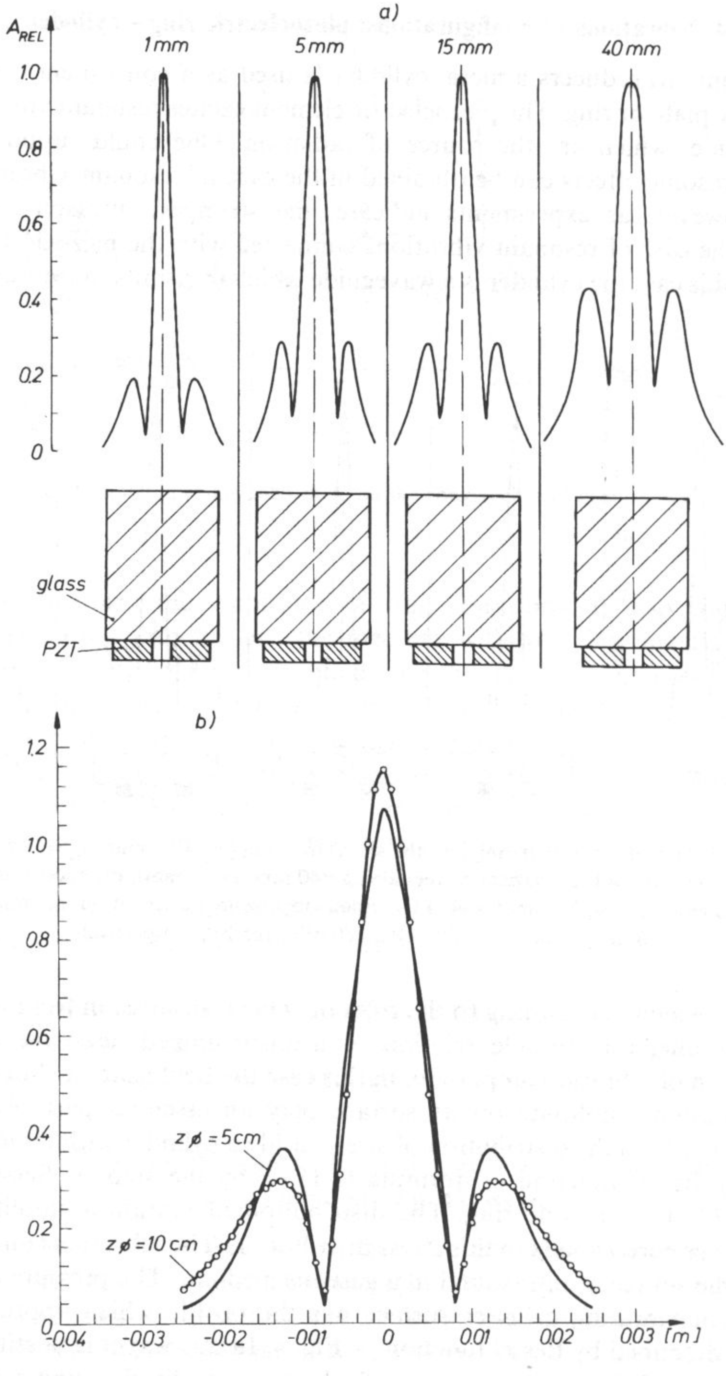


Fig. 4. Acoustic field generated in the air by vibrating piezoelectric ring ($\phi_{ext}=26$ mm, $\phi_{int}=5$ mm) with cylindrical waveguide ($\phi=30$ mm, $l=40$ mm) at different distances from the cylinder surface, $f=206$ kHz. a) measured using acoustic probe, b) calculated for truncated Bessel function distribution.

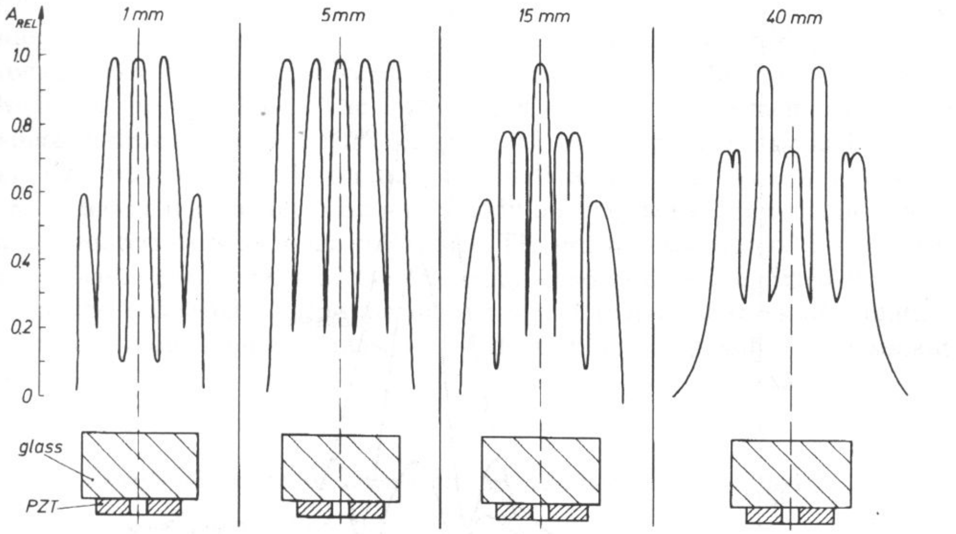


Fig. 5. Distribution of acoustic pressure in the air in the vicinity of cylindrical waveguide ($\phi=35$ mm, $l=20$ mm) excited to vibrations by the piezoelectric ring ($\phi_{ext}=26$ mm, $\phi_{int}=5$ mm), distances from the cylinder surface: 1 mm, 5 mm, 15 mm, 40 mm; $f=195$ kHz.

waveguide materials it is possible to construct transducers with better acoustic matching to the medium. However as the measurements indicate (Fig. 5) there are vibration amplitude distributions on the waveguide surface different from zero-order Bessel function. This causes the deformation of the directivity patterns and the increase of the beam diffraction.

5. Influences of additional Bessel and Gauss functions on the directivity characteristics

Applying the transfer function method we have made a series of computer simulations for the examination of effects which occur when the distribution of vibration amplitude differs from Bessel function. The calculations have shown that the presence of additional functions in the vibration distribution on the cylinder surface has the unfavourable influence. It causes the beam broadening and the limitation of the nondiffracting beam range. The results indicate that the elimination of these disturbing functions is necessary. The search of ways of their limitation is important but difficult.

As an example the results of calculations are presented in Fig. 6 for Bessel distribution $J_0(\alpha r)$, Gauss distribution $\exp[-(r/b)^2]$ and the product $\exp[-(r/b)^2]\cdot J_0(\alpha r)$. The results indicate that the presence of Gauss function is

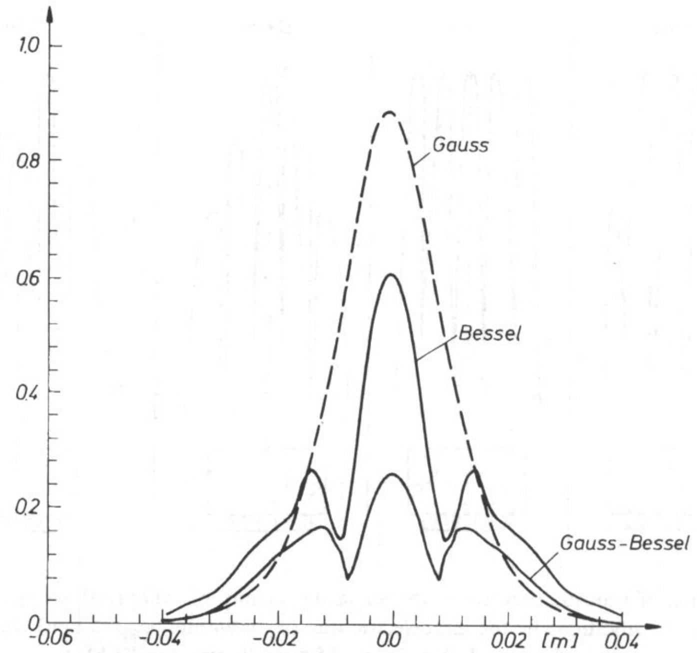


Fig. 6. Influence of the vibrations disturbing Bessel distribution on the radiation characteristics of the ring, $f=334$ kHz, distance 30 cm.

unfavourable. The situation is similar for the sum or product of Bessel functions with different arguments. The characteristics of radiation are worse than for pure zero-order Bessel function. Thus the complication of the vibration distribution does not lead to the wanted radiation characteristic.

6. Problem of the frequency characteristic

Used methods of acoustic field generation with resonant piezoelectric transducers coupled to elastic elements (cylinder, plate, etc.) have the important shortcoming — narrow frequency band. The vibration amplitude distribution on the transducer surface changes with a change of frequency. This causes changes of the directivity pattern. Theoretical methods of acoustic field calculation permit to solve the problem for wide frequency band, however the practical realization of such systems with high efficiency is difficult. It leads to the complex systems with coupled vibrations or with strongly damped vibrations and even to the work of piezoelectric element beyond its resonance, what strongly limits the efficiency.

The width of frequency band of a transducer depends on the electromechanical coupling coefficient. Different modes have different electromechanical coupling coefficients, then different frequency bandwidth. The introducing of additional vibrating elements (in the form of cylinder or plate) causes the decrease of electromechanical coupling coefficient. It leads finally to the reduction of frequency bandwidth of the transducer. The introduction of an elastic element causes the unfavourable distribution of elastic energy at the cost of piezoelectric and electric energy. One obtains wanted vibration distributions but the frequency band is reduced. It means that the introduction of additional elements is not advantageous for the transducers working in the pulse mode. The practical solution of this problem is very difficult. The keeping of the frequency bandwidth is connected with the change of the wavelength in the medium, it would require changes of vibration amplitude distribution on the transducer surface. It is not possible for resonant vibrations.

7. Results of the measurements of ring radiation through cylinders

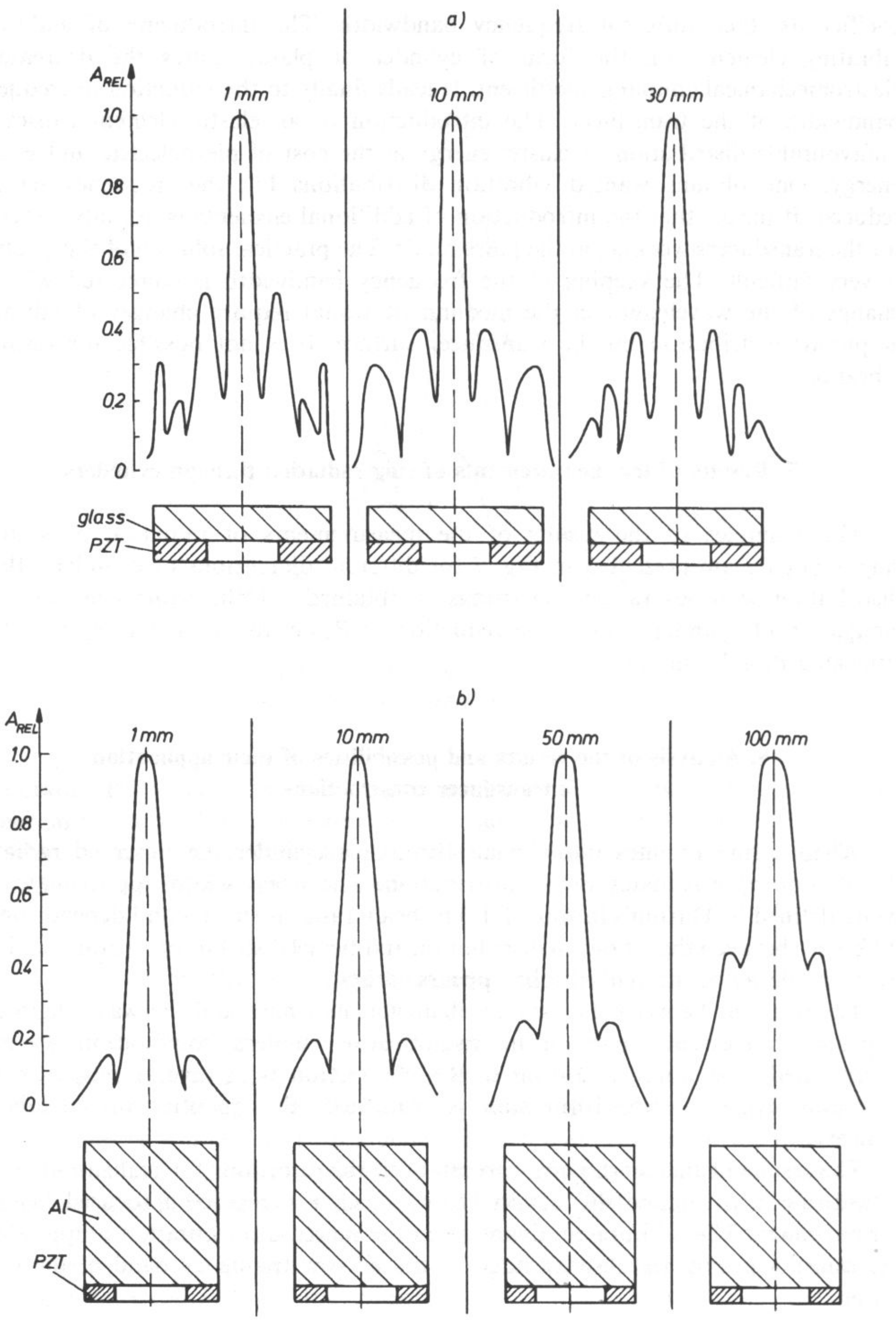
The examples of the results of the measurements of radiation of systems: ring – cylinder are presented in Fig. 7 for different dimensions of cylinders. Bessel distribution with several zero-crossings is obtained for the short cylinders. The elongation of cylinders causes the reduction of Bessel functions terms, it leads to truncated Bessel function.

8. Analysis of the results and possibilities of their application in transducer constructions

When a ring radiates into the air through a cylinder the observed radiation distribution approximates Bessel distribution. The beam width approximates the width defined by Durnin's formula [5]. The beam range is shorter and depends on the difference between the vibration distribution and Bessel distribution. Strong sidelobes exist, the decay of the central lobe appears earlier.

The relations between the cylinder diameter, its length and the wavelength play apparently the essential part. In the case of short cylinders the vibrations are more complicated, one obtains zero-order Bessel function with several zero-crossings. For long cylinders Bessel function is truncated, the quantity of sidelobes is reduced.

The results of this work permit to establish the directions for realizations of the transducers with limited diffraction beam, which can replace a focused beam in certain range. The additional advantage is the increase of vibration amplitude on the transducer axis, the disadvantage — the narrow frequency band of the transducer.


a)
A_REL
1.0
0.8
0.6
0.4
0.2
0
1 mm
10 mm
30 mm
glass
PZT
b)
A_REL
1.0
0.8
0.6
0.4
0.2
0
1 mm
10 mm
50 mm
100 mm
Al
PZT

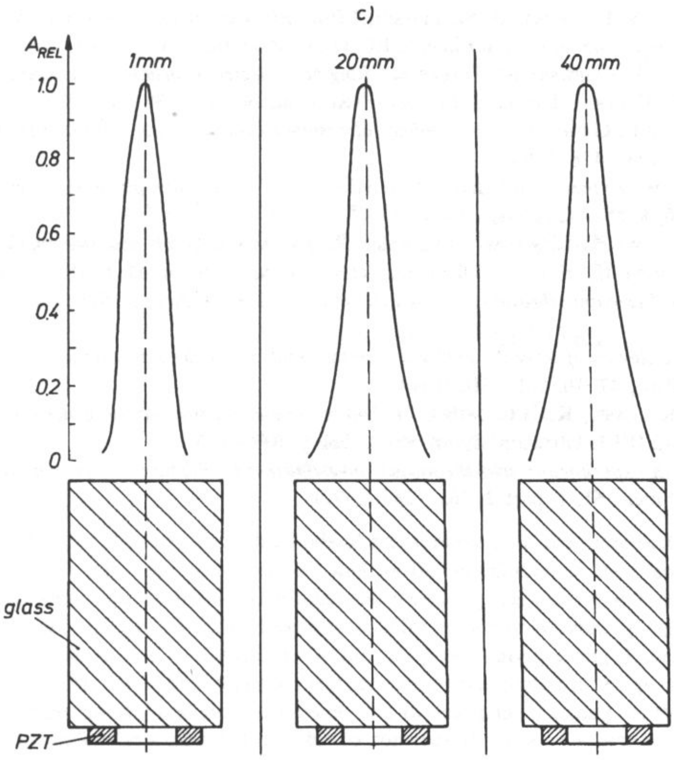


Fig. 7. Examples of acoustic fields in the air generated by PZT rings with different waveguides. a) glass waveguide, $l=10$ mm, $\phi=50$ mm; ring: $\phi_{ext}=50$ mm, $\phi_{int}=20$ mm; $f=252$ kHz, b) Al waveguide, $l=40$ mm, $\phi=38$ mm; ring: $\phi_{ext}=38$ mm, $\phi_{int}=20$ mm; $f=206$ kHz, c) glass waveguide, $l=65$ mm, $\phi=40$ mm; ring: $\phi_{ext}=30$ mm, $\phi_{int}=16$ mm; $f=194$ kHz.

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