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ACOUSTIC MODELLING OF SURFACE SOURCES PART II. PISTON MODEL, ASYMMETRICAL FUNCTION OF VIBRATION VELOCITY, DISCRETIZING ERROR, AXISYMMETRICAL PROBLEM

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In the paper, by applying irregular discretization of the axisymmetrical plane source, an optimal piston model was obtained. An asymmetrical function of the vibration velocity, with regard to its zeros was assumed on vibrating surface. The vibration velocity of the single piston was calculated as an integral mean value of the vibration velocity of the vibrating surface. Then the model of the source consists of an array of driving pistons. In order to select the optimal position of the boundary between the pistons, the measure of the divergence/convergence between directivity function of the model and exact was analysed. The least squares distance and uniform distance were taken as a measure of deviation of the directivity function of the model from the exact one. Numerical calculations showed that the irregular discretization gives particularly good results for asymmetrical function of vibration velocity. It was also proved that the least squares distance is a better measure of directivity functions of the model and exact one the uniform distance. Its variation, as a function of vibration velocity or position of the vibrating surface in the baffle or nondimensional wave number, is more regular than the variation of uniform distance.

1. Introduction

Modelling acoustical source with arbitrary geometry is the subject of a lot of articles, e.g. paper [5]; see also Refs. given in [5]. Mathematical model of such sources contains, among other things, boundary integral equations (BIE). Boundary element method (BEM) is applied to solve BIE. The first step of the modelling is discretization. Both geometry surface of the source and the acoustic variables are discretized.

In literature cited in [5], sources with arbitrary geometry were considered, however constant, acoustic variables were assumed on the whole surface. In these papers, planar elements of the surface were utilized, and in addition, the acoustic variables were represented by the constant value on each element. In such case, discretization applies mainly to geometry. Primary discretization is imposed by the singularities of the surface, e.g. sharp edges. More refined model may be obtained if the elements of primary discretization are discretized again [4, 5, 6].

In paper [3] a planar source was examined, but vibration velocity function not constant was assumed. In this instance, vibration velocity is discretized, but not the surface of the source. Furthermore, in Ref. [3], general rules of primary discretization of vibration velocity with nodal lines were given. In Ref. [3] as in Refs. mentioned above, the secondary regular discretization was applied to improve the quality of the model. As expected, if the number of regular elements was increased very good piston model was obtained. However, it contains too many elements. Using too many elements is a burden to the user and is also not computationally efficient.

In order to partially remedy the difficulty, irregular discretization was suggested, Ref. [2]. As an example, planar axisymmetrical source was considered which creates axisymmetrical surface of the vibration velocity. Therefore, due to the symmetry of the surface, only its cross section line has to be discretized. As it was pointed out in Ref. [2], the irregular discretization gave a good irregular model which contained not so many elements as the regular model. The irregular model which contained the minimum number of the elements and whose acoustical field fulfilled the assumed boundary convergence to the exact acoustical field was called optimal model.

In Ref. 2, sinus function was chosen as the function of vibration velocity. This function has the maximum halfway along the distance between zero points, i.e. it is symmetric to its maximum. The absolute distance (in dB) was assumed as the measure of the convergence.

In this paper, a planar axisymmetrical source was considered. But the shape of the vibration velocity is not symmetric to its maximum. Two new measures of the convergence were taken into account, namely (1) the least squares distance and (2) uniform distance nondimensional between exact directivity function and the directivity of the model.

In the paper two problems were solved. First, the influence of asymmetry of the vibration velocity, frequency of the vibration, place of the vibrating surface in the plane baffle on the optimal piston was found. Secondly, it was pointed out that, generally, the least square distance is a better measure of convergence between directivity functions than the uniform one.

2. Acoustic field of the axisymmetric (AS) source

In this and the next section, an outline of the theory is presented upon which the calculations are based. A detailed treatment of the theory can be found in Ref. [2].

The acoustic field of the vibrating surface placed in an infinite plane and rigid baffle is given by Huygens–Rayleigh integral. Assuming fully AS source (i.e. both the source shape and acoustic variables are independent of the angle of revolution of the source), an acoustic field is AS too, and Huygens–Rayleigh integral takes the form [2]

$$\Phi_{0N}=\frac{1}{2\pi}\int_{\rho} v_{0N}(\rho)\left[\int_{0}^{2\pi} G(r)\,\mathrm{d}\phi\right]\rho\,\mathrm{d}\rho, \tag{2.1}$$

where

$$G(r)=\frac{\exp[-\mathrm{i}kr(\rho,\phi)]}{r(\rho,\phi)}, \tag{2.2}$$

$$v_{0N}(\rho)=C_1\sin(\pi\rho)\exp(-C_2\rho), \tag{2.3}$$

k — nondimensional wave number, $v_{0N}(\rho)$ — vibration velocity of the driving surface asymmetrical to its maximum, C_1, C_2 — coefficients, they have been selected to fulfil the assumed asymmetry of the $v_{0N}(\rho)$. General geometry of the radiation problem is given by Fig. 1.

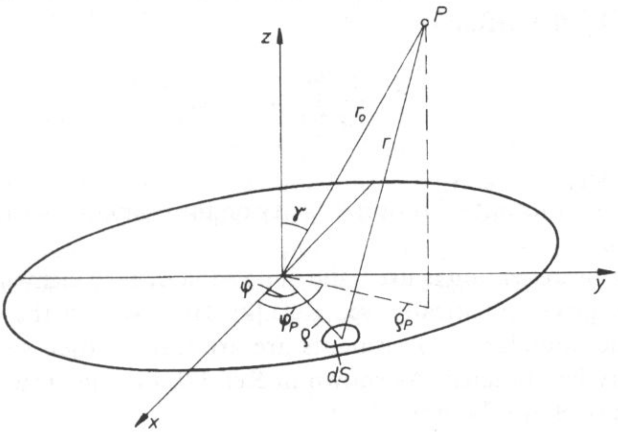

Fig. 1. Nomenclature for a radiation problem.

In the far distance away from the source, the Fraunhofer approximation is applied (Ref. [2]).

$$\frac{1}{r}\cong\frac{1}{r_0}, \qquad r\cong r_0-\rho\cos(\rho,r_0)=r_0-\rho\sin\gamma\cos(\phi-\phi_P). \tag{2.4}$$

Substituting Eq. (2.4) into Eq. (2.1) we obtain

$$\Phi_{0N}(\gamma)=C_0\,Q_{0N}(\gamma), \tag{2.5}$$

where

$$C_0=\frac{\exp(-\mathrm{i}kr_0)}{r_0}, \tag{2.6}$$

$$Q_{0N}(\gamma)=\int_{\rho} v_{0N}(\rho)\,J_0(k\rho\sin\gamma)\rho\,\mathrm{d}\rho, \tag{2.7}$$

$J_0(x)$ — Bessel function of the first kind and zeroth order.

In a great distance from the source, the directivity function is usually analyzed. To obtain directivity function the acoustic field Eq. (2.5) may be normalized by the acoustic field Eq. (2.6), generated by the point source set in the origin of the coordinates. Then, under these circumstances, the directivity function is given by (2.3).

3. Acoustic field of the piston model

The mean vibration velocity

The constant vibration velocity on *j*-element as the mean velocity — v_j^m over the driving surface of the source has been calculated. If AS case is considered, the mean velocity is given by the definition

$$v_j^m = \frac{1}{L_j} \int_{G_{\rho 1}}^{G_{\rho 2}} v_{0\mathrm{N}}(\rho)\, \mathrm{d}\rho, \tag{3.1}$$

where $L_j = G_{\rho 2} - G_{\rho 1}$.
The model with the constant vibration velocity on the elements of the surface is called the piston model.

If the regular discretization is used, the dimension of each element is the same and $G_{\rho 1}$, $G_{\rho 2}$ may be given analitically, see Ref. [2]. However, in the case of irregular discretization, the boundaries of elements are arbitrary chosen so that an optimal piston model may be obtained. As shown in Ref. [2], the dimensions of elements in optimal piston model are different.

If Eq. (2.6) is substituted into Eq. (2.7), the directivity function of the piston model is obtained

$$Q_{0\mathrm{N}}^m(\gamma) = \sum_{j=1}^{J} v_j^m \int_{\rho_j} J_0(\mathrm{k}\rho \sin\gamma)\rho\, \mathrm{d}\rho. \tag{3.2}$$

4. Verification of the model

The quality of the piston model may be improved in two ways if:
1 — the discretization is increased,
2 — assuming that the number of pistons is constant the dimensions of the pistons are changed.
To simplify the notation, the following labels are introduced:
M_r — regular piston model,
M_i — irregular piston model.
First of them consists of J equal elements and the second contains J different elements.

To analyse the position of directivity functions in relation to each other (convergence/divergence of the directivity functions) optimal methods may be used.

Then, the optimal model ought to be built. The main problem of optimalization is the minimization or the maximization of some function subjected to some constraints. The function whose the least or the greatest value is being sought is called an objective function and the collection of the values of the variables at which the least value is attained defines the optimal solution.

To simplify an optimalization analysis, two element model ($J=2$) is chosen. Assume that the constraints are given by

$$r_\alpha - G_1 \geqslant 0, \quad G_1 \geqslant 0, \tag{4.1}$$

where

$$G_1 = G.$$

It is required to choose among the set of G such G that would minimize the objective function.

The objective function may be:

- least squares distance between $Q_{0N}(\gamma)$ and $Q_{0N}^m(\gamma,G)$ or
- uniform distance among the points of $Q_{0N}(\gamma)$ and $Q_{0N}^m(\gamma,G)$.

The directivity function $Q_{0N}^m(\gamma,G)$ is a function of G, Eqs. (3.1) and (3.2), in which G appears in boundaries of integrals. Then, the optimalization belongs to nonlinear optimalization. Applying one of the methods of solving nonlinear optimization model (Ref. [7]), an optimal solution is obtained. The model which is connected with an optimal solution may be called optimal model.

4.1. *Least squares distance between directivity functions*

Optimalization of least squares distance is closely related to the least squares method [1].

It is necessary to approximate the function $Q_{0N}(\gamma)$, i.e. the exact directivity function, with the function $Q_{0N}^m(\gamma,G)$, i.e. the directivity function of the model. An approximate function $Q_{0N}^m(\gamma,G)$ may be written

$$Q_{0N}^m(\gamma,G) = \sum_{j=1}^{J} v_j^m(G) f_j(\gamma,G),$$

where the base functions are

$$f_j(\gamma,G) = \int_{\rho_j} J_0(k\rho \sin\gamma)\rho \, d\rho.$$

The number

$$d_l(Q_{0N}, Q_{0N}^m) = d_l(G) = \left\{ \int_0^{\pi/2} [Q_{0N}(\gamma) - Q_{0N}^m(\gamma,G)]^2 \, d\gamma \right\}^{1/2}, \tag{4.4}$$

is called the least squares distance between functions $Q_{0N}(\gamma)$ and $Q_{0N}^m(\gamma,G)$.

The main problem of the least mean method is to find such a function $Q_{0N}^{m}(\gamma,G)$ so that the least squares distance ought to be minimal. Thus, the main aim is to find the function Q_{0N}^{m}, (γ, G).

In fact, suitable coefficients $v_j^m(G)$ are looked for. This means that suitable vibration velocity on the elements are looked for. Analysing the least squares distance $d_l(G)$ between directivity functions, the acoustical field of the model is investigated directly, however the vibration velocity on the elements of the model is investigated indirectly.

The problem mentioned above may be solved as optimalization problem, i.e. the minimum of the $d_l(G)$ is searched without regard to function $Q_{0N}^{m}(\gamma,G)$. As a result, an optimal model is obtained which is the best approximation of the source in terms of the least squares distance between directivity functions.

4.2. *Uniform distance between directivity functions*

Optimalization of the model may be carried out if the uniform distance between directivity function as an objective function is chosen. The objective function is the distance $d_u(\gamma,G)$ between some point $A_0(x_0,y_0)$ of one directivity function and the straight line which passes through neighbouring points $A_1(x_1,y_1)$, $A_2(x_2,y_2)$ of the other directivity function, Fig. 2. The distance $d_u(\gamma,G)$ may be accepted as the measure

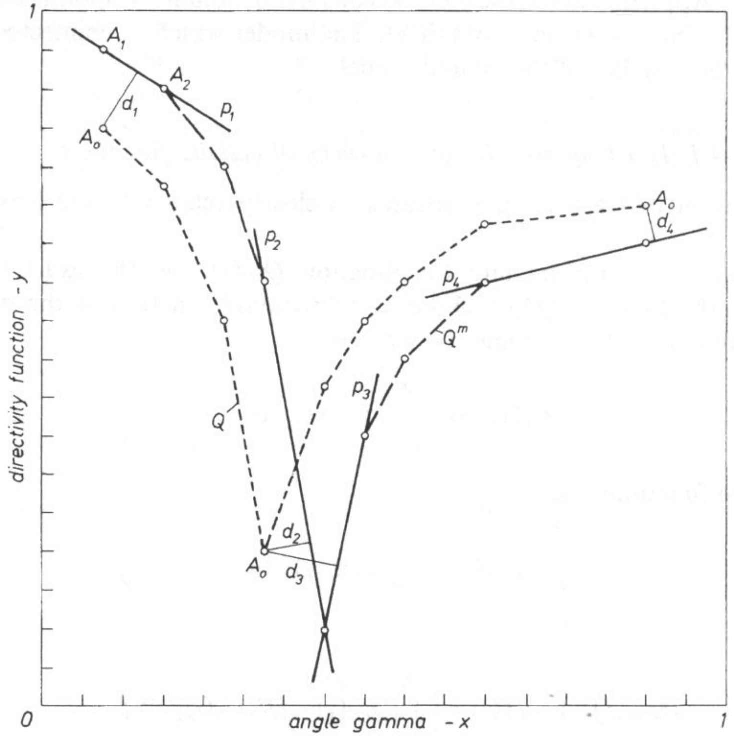

Fig. 2. Uniform distance d_u between the lines as a function of angle gamma.

of the distance between directivity functions at point A_0. Let point B be a projection on the straight line A_1A_2 of the $A_0(x_0, y_0)$. This measure will be the best if the projection of B is the midpoint between points A_1, A_2. The distance $d_u(\gamma,G)$ can be expressed in terms of the coordinates of the points A_0, A_1, A_2 by the formula

$$d_u(\gamma,G)=\frac{a_1x_0+a_2y_0+a_3}{\sqrt{a_1^2+a_2^2}}, \tag{4.5}$$

where

$$a_1=y_1-y_2, \quad a_2=x_2-x_1, \quad a_3=x_1y_2-y_1x_2.$$

It should be noted, that if the acoustical field of AS source is analyzed, the acoustical pressure [Pa] or the directivity function [dB] are displayed along the axis of abscissa. Therefore, the concept of the distance between directivity functions introduced above is meaningless for two reasons. First, a unit of the distance $d_u(\gamma,G)$ has no physical sense. Second, because of the different scales of the axis the influence of the values of ordinates and abscissas on the value of the distance $d_u(\gamma,G)$ will be different. This difference is particularly visible if the directivity functions are given in dB scales considered.

In order to overcome these difficulties the directivity functions ought to be written in normalized coordinates. These coordinates are nondimensional and they are transformed so that they may take values $\langle 0,1 \rangle$.

For the directivity functions of which the ordinates and abcissas vary between 0 and 1 the nonlinear programming methods point out the boundary G of optimal two-piston model (M_0, $J=2$).

Applying nonlinear programming methods to such complicated function as (4.4) or (4.5) is very difficult [7]. It is convenient to calculate the value of objective function for the discrete values of G_v and next to chose that G for which the function possesses a minimum.

If the least squares distance is the objective function, then for the set of G_v, the set of values $d_{l,v}$ is calculated; the minimal value $d_{l,\min}$ points out the boundary $G_{0,l}$ of the optimal model.

By using the uniform distance as an objective functions the searching for the boundary G_{0u} of optimal model overlooking nonlinear programming method is fairly difficult. It can be done by using the following approach described below. For the constant G_v a finite number of μ points $A_0(\gamma_\mu)$ of the directivity function Q_{0N} is determined. For each point $A_0(\gamma_\mu)$ the distance $d_u(\gamma_\mu,G_v)=d_{u,v}(\gamma_\mu)$ from the directivity function $Q_{0N}^m(\gamma,G_v)$ is searched. In this way the set of values $d_{u,v}(\gamma_\mu)$ is obtained from which the maximum of $d_{u,m}$ suitable to G_v ought to be chosen. This procedure outlined above ought to be repeated for the set of G_v. From the set of $d_{u,m}$ the maximum value of $d_{u,\min}$ is chosen which is pointing out G_{ou} of the optimal model. Using the uniform distance as a measure of divergence of the acoustic field has such an advantage that for constant G_v it is possible to investigate $d_{u,v}(\gamma_\mu)$ as a function of the angle γ.

It is obvious that for separate values of G_v, regions of three dimensions space described by angle γ_μ can be found where the divergence of directivity functions is the biggest. As shown in Fig. 2 the biggest divergence is at minimum or maximum and it is equal $d_{u,m}=d_3>d_1>d_2>d_4$. It is not necessary to investigate $d_{u,v}(\gamma_\mu)$ the full interval of the angle γ. Only maxima or minima of the directivity functions pointed out $d_{u,v}(\gamma_\mu)$. The jump of discontinuity from d_2 to d_3 should be noted.

5. Numerical calculations

The numerical calculations were performed to solve two basic problems: 1 — to establish the influence of the asymmetry of the vibration velocity function, frequency of vibration and the place of driving surface in the baffle on the optimal model, 2 — to point out a better method of investigation of the convergence (divergence) between directivity function of the model and exact one. Furthermore it was shown, assuming asymmetrical function of vibration velocity, that the irregular piston model gives a better result then the regular one if the number of elements in the both models is the same. To simplify the numerical calculations a two-elements model was considered, i.e. $J=2$. Further in the paper, to simplify the notation, the subscript J has been dropped out.

The aim of the first calculation is to show which measures of divergence should be optimized to obtain a better optimal model. These calculations consist of three steps marked **I.1**, **I.2**, **I.3**.

I.1. For fixed position of the vibrating surface in a baffle, $a=0$, $b=1$ and $k=2.5$, the place of maximum of the vibration velocity $v_{\max}\,\rho_c$, was changed. For $\rho_c=\{.3, .4, .5, .6, .7\}$ the detailed results were presented in Figs. 3, 4, 5, 6, 7 respectively.

Each figure consists of part A and B.

Part A presents:

1 — a picture of vibrating surface,

2 — plots:

— least squares distance $d_{i,v}$

— uniform distance $d_{u,v}$

— table which explains the symbols.

Part B presents:

1 — cross-section of vibrating surface and M_{ol} which is obtained as a result of optimalization of d_l,

2 — directivity functions:

— exact (solid line),

— of the model M_r,

— of the model M_{ol},

— of the model M_{ou} which is obtained as a result of optimalization of d_u,

— table which explains the symbols.

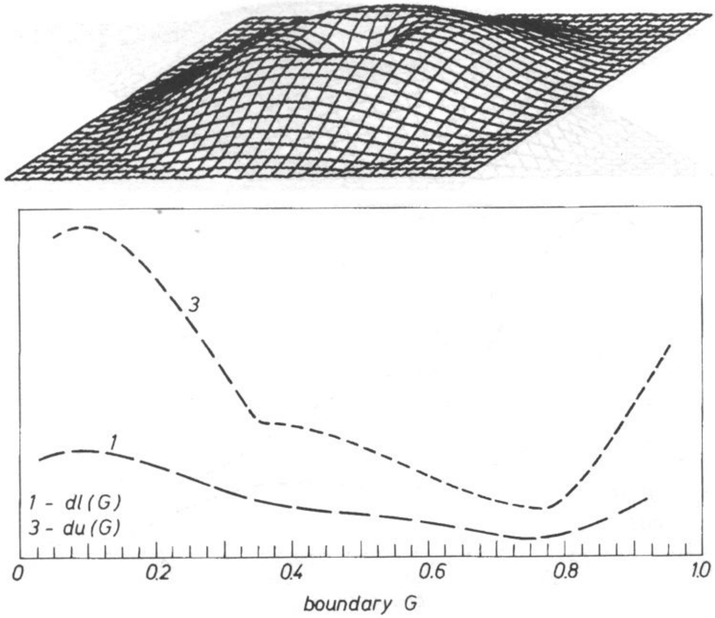

Fig. 3A. Picture of the vibrating surface, $\rho_c = 3$. Plots of the mean square distance $d_{l,v}$ and relative distance $d_{u,v}$ $a=0$, $k=2.5$.

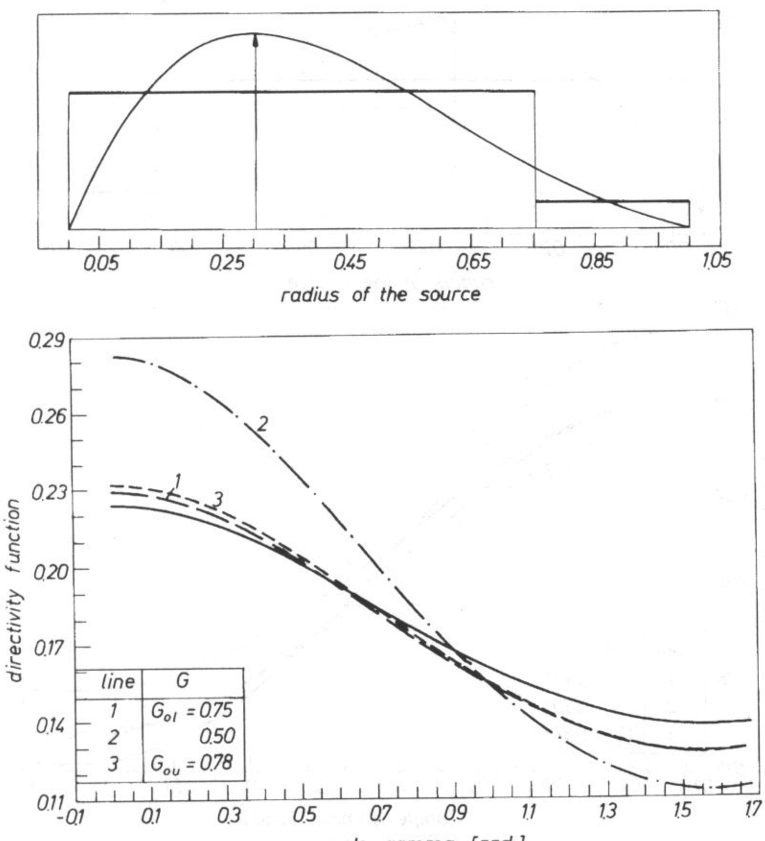

Fig. 3B. Cross-section of the vibrating surface ($\rho_c = 3$) and M_{ol}. Directivity functions of the source M_{ol}, M_r and M_{ou}; $a=0$, $k=2.5$.

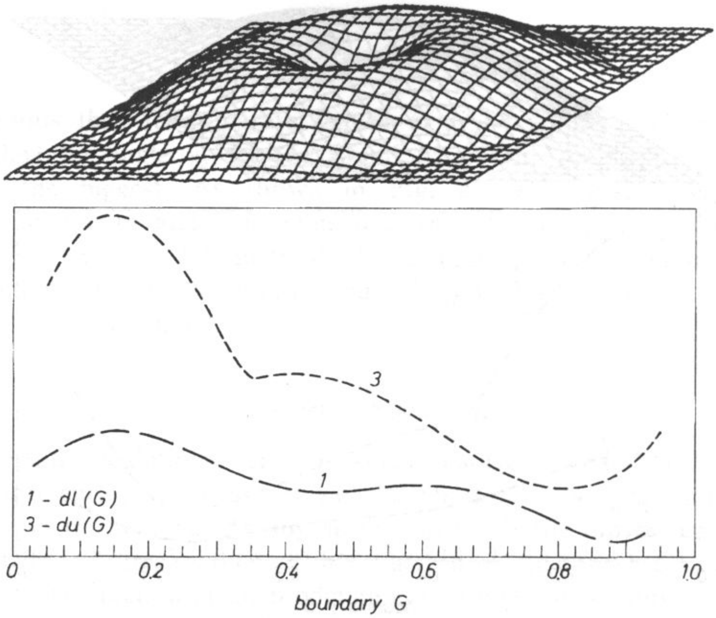

Fig. 4A. Picture of the vibrating surface $\rho_c = .4$. Plots of the mean square distance $d_{l,v}$ and relative distance $d_{u,v}$, $a=0$, $k=2.5$.

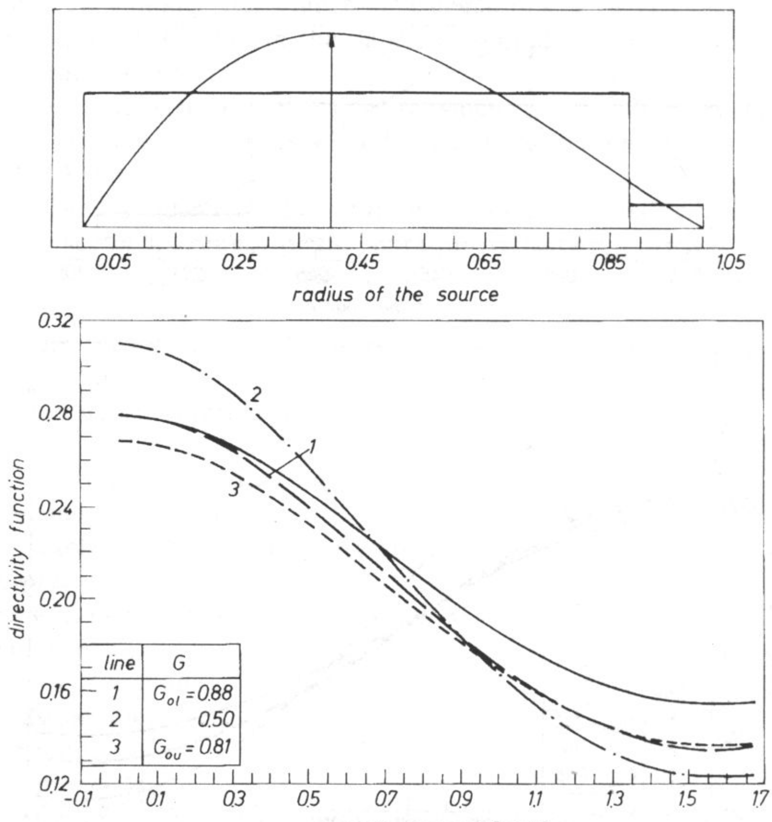

Fig. 4B. Cross-sections of the vibrating surface ($\rho_c = .4$) and M_{ol}. Directivity functions of the source, M_{ol}, M_r and M_{ou}; $a=0$, $k=2.5$

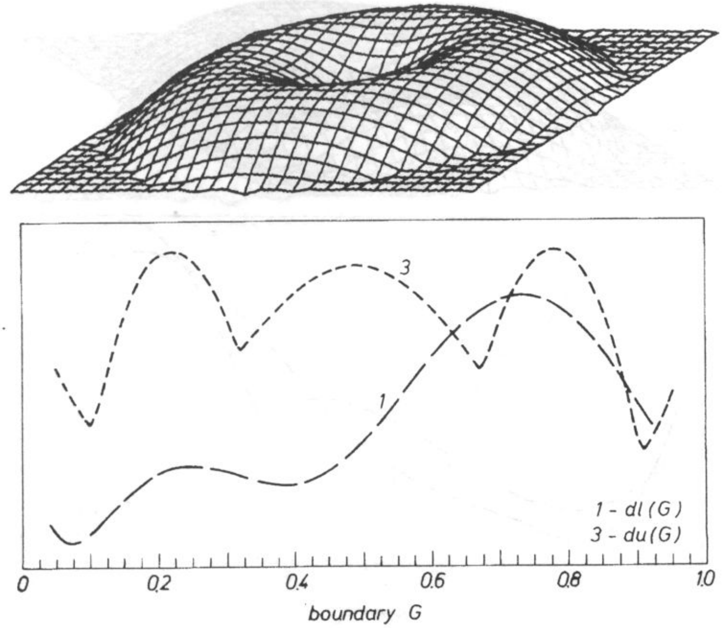

Fig. 5A. Picture of the vibrating surface $\rho_c = .5$. Plots of the mean square distance $d_{l,v}$ and relative distance $d_{u,v}$, $a=0$, $k=2.5$.

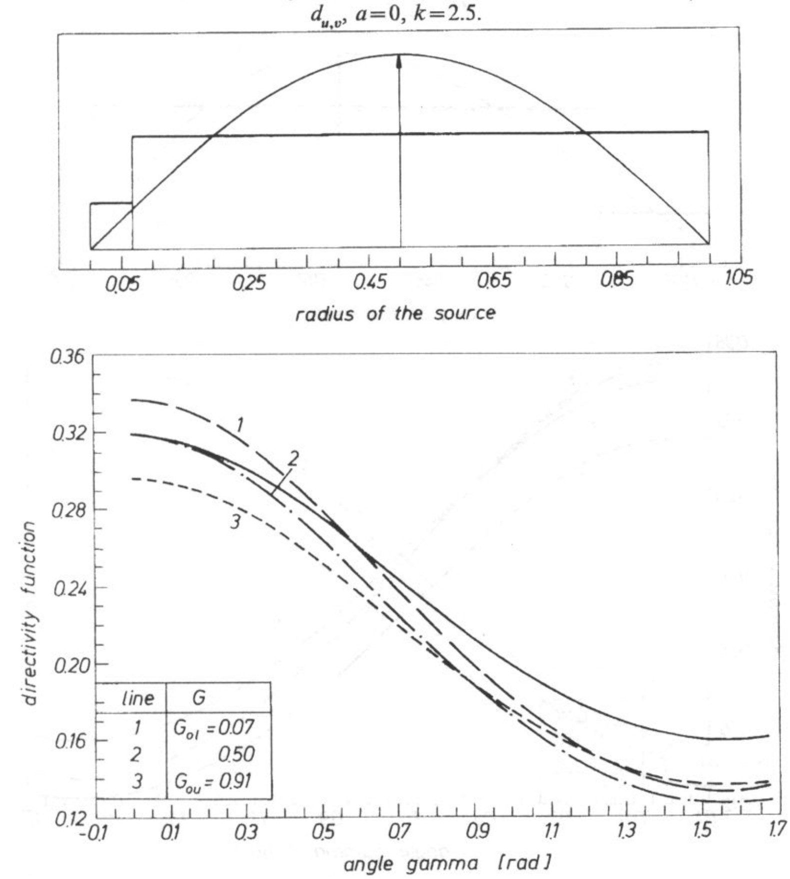

Fig. 5B. Cross-sections of the vibrating surface ($\rho_c = .5$) and M_{ol}. Directivity functions of the source, M_{ol}, M_r and M_{ou}; $a=0$, $k=2.5$.

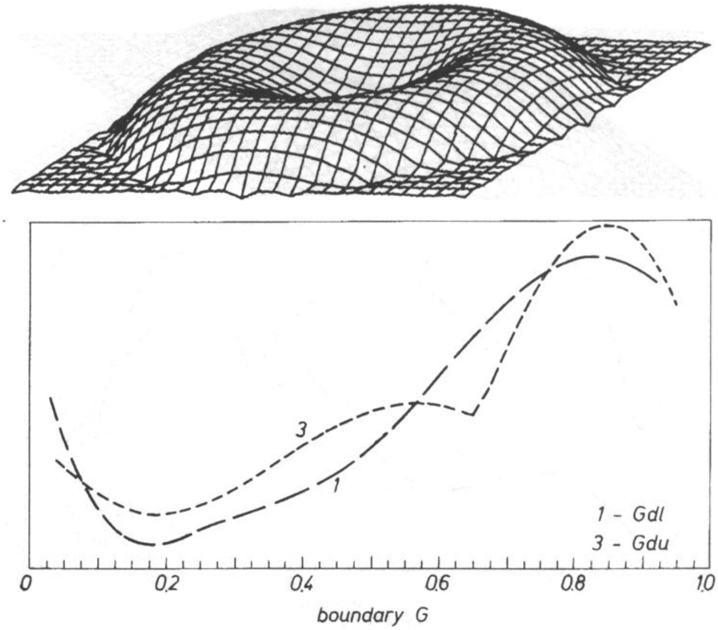

Fig. 6A. Picture of the vibrating surface $\rho_c = .6$. Plots of the mean square distance $d_{l,v}$ and relative distance $d_{u,v}$, $a=0$, $k=2.5$.

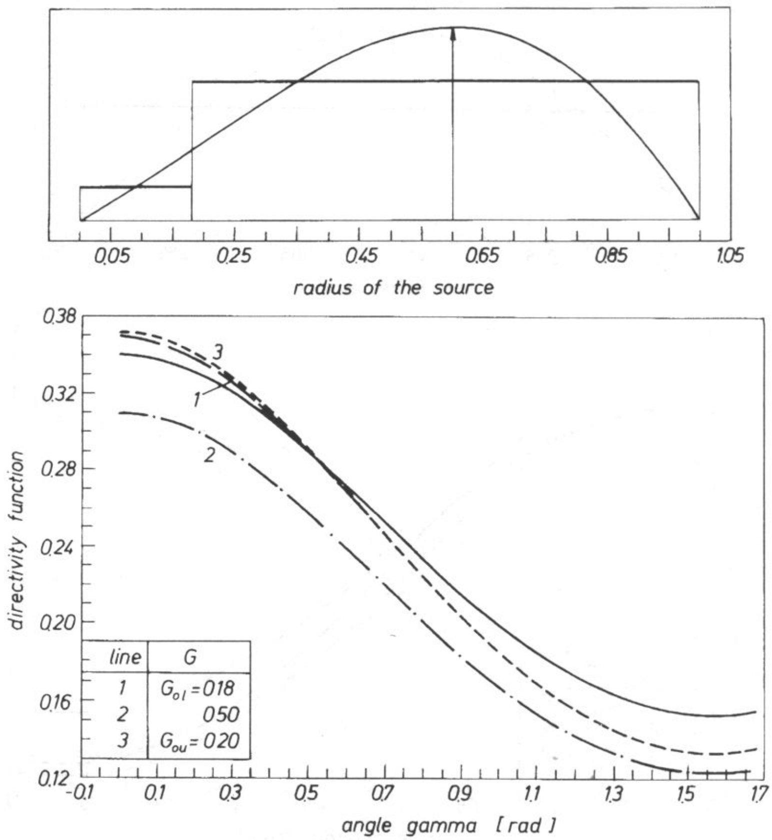

Fig. 6B. Cross-sections of the vibrating surface ($\rho_c = .6$) and M_{ol}. Directivity functions of the source, M_{ol}, M_r and M_{ou}; $a=0$, $k=2.5$.

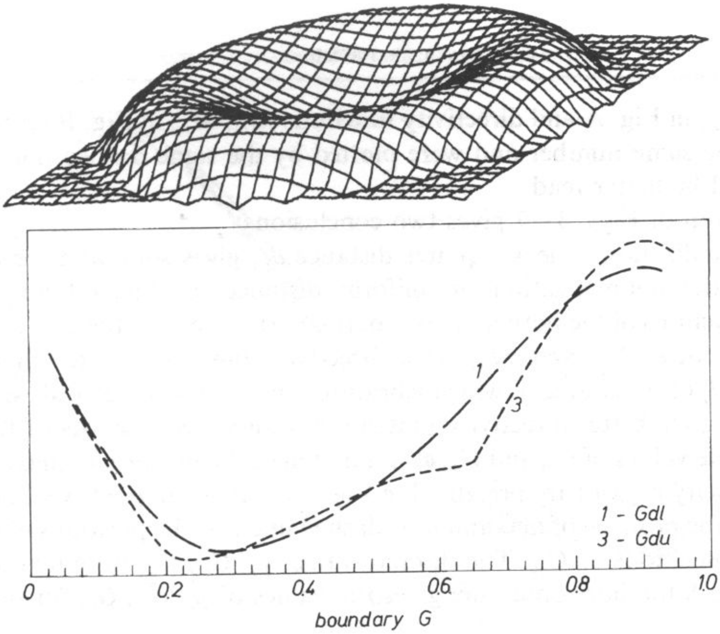

Fig. 7A. Picture of the vibrating surface $\rho_c = .7$. Plots of the mean square distance $d_{l,v}$ and relative distance $d_{u,v}$, $a=0$, $k=2.5$.

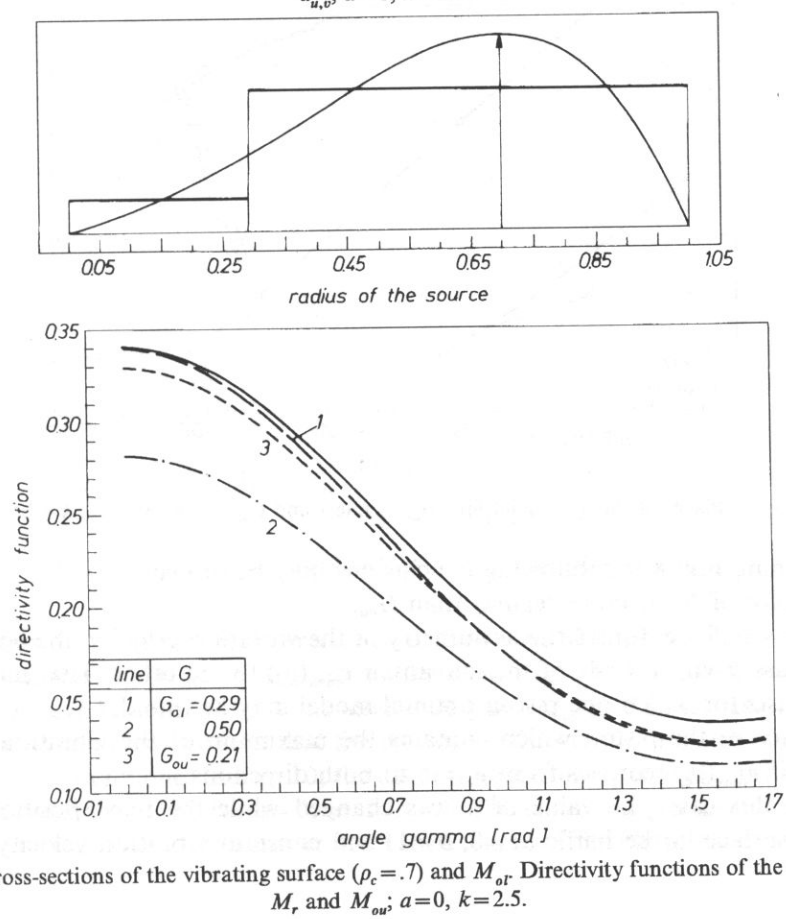

Fig. 7B. Cross-sections of the vibrating surface ($\rho_c = .7$) and M_{ol}. Directivity functions of the source, M_{ol}, M_r and M_{ou}; $a=0$, $k=2.5$.

Curves $d_{l,v}$, $d_{u,v}$ in Fig. A and directivity functions M_{ol}, M_{ou} in Fig. B respectively are marked by the same number and were plotted by the same dashed line so that the figures should be better read.

Examination of Figs. 3—7 gives two conclusions:

1 — optimalization of least squares distance $d_{l,v}$ gives somewhat better optimal model than the optimalization of uniform distance d_u. The difference between directivity functions of the both models is particularly seen near the axis of the source. 2 — both models, M_{ol} and M_{ou}, give directivity functions better than M_r. The exception is M_r of the source of which vibrating velocity is symmetrical, see Fig. 5 line 2, and which gives better directivity function but only near the axis of the source.

The discrete values of G_{ol} and G_{ou} as a function of the position of maximum of the vibration velocity $v_{max}(\rho_c)$ are presented in Fig. 8 where $a=0$, $b=1$, $k=2.5$. The solid line indicates the position of maximum ρ_c, dashed line 1 — the positions of G_{ol}, dashed line 3 — the positions of G_{ou}. The discrete values ρ_c, G_{ol}, G_{ou} were marked by light points. In Fig. 8 the horizontal line gives the values of ρ_c, G_{ol}, G_{ou} for one source.

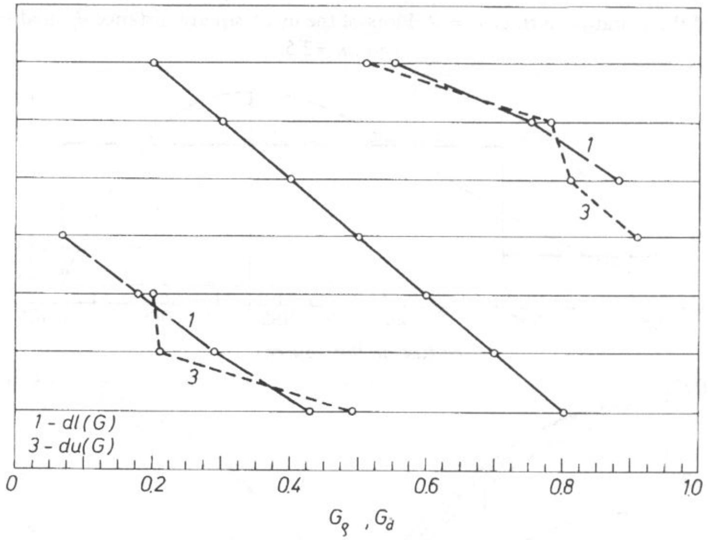

Fig. 8. Places of the ρ_c — solid line, G_{ol} — line 1 and G_{ou} — line 3; $a=0$, $k=2.5$.

Examining Fig. 8 the following conclusions may be drawn:

• the plot of G_{ol} is more regular than G_{ou},

• near $\rho_c \simeq .5$, i.e. for a little asymmetry of the vibration velocity, the boundaries G_{ol}, G_{ou} pass from one side of the maximum $v_{max}(\rho_c)$ to the other. That means that such ρ_c exists for which one-piston optimal model may be found.

• surface of the piston which contains the maximum of the vibration velocity decreases if $v_{max}(\rho_c)$ removes from $\rho_c=.5$ to both directions.

I.2. In this case, the value of k was changed while the fixed position of the vibrating surface in the baffle ($a=0$, $b=1$) and constant vibration velocity ($\rho_c=.3$)

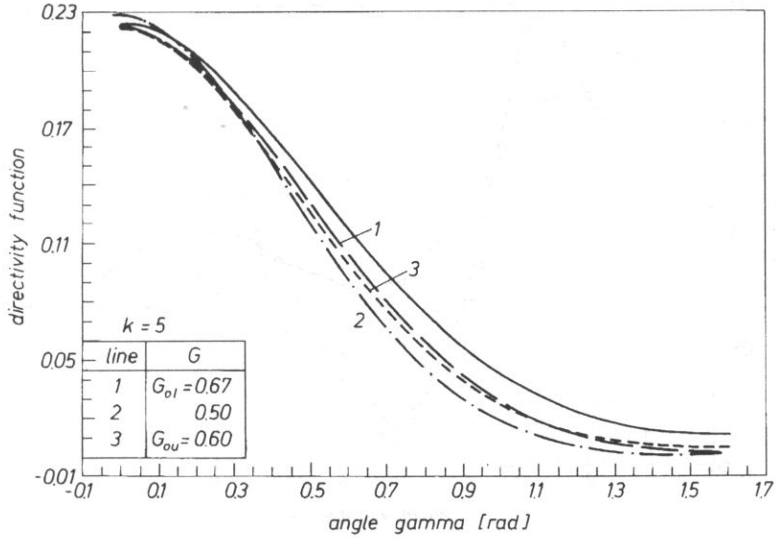

Fig. 9A. Directivity functions of the source, M_{ol}, M_r and M_{ou}; $k=5$, $a=0$, $b=1$, $\rho_c=.3$.

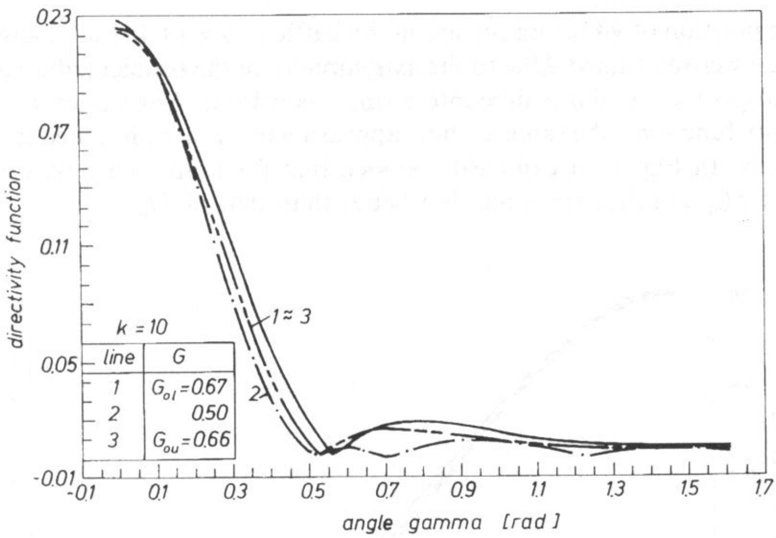

Fig. 9B. Directivity functions of the source, M_{ol}, M_r and M_{ou}; $k=10$, $a=0$, $b=1$, $\rho_c=.3$.

were assumed. The analytical directivity function (solid line) and next directivity functions of the M_r, M_{ol} and M_{ou} are displayed in Fig. 9A, B for discrete values of $k=5$, 10. As can be seen in Fig. 9 the models M_{ol} and M_{ou} give better results than M_r. In Fig. 9B, it can easily be seen that the plot of the directivity function of M_u is even other than the exact one. In Fig. 10, the discrete values of G_{ol} and G_{ou} as functions of wave number k are shown for $a=0$, $b=1$, $\rho_c=3$. The curve of G_{ol} is more regular than the curve of G_{ou}. For wave number $k>7$ these curves coincide to each other.

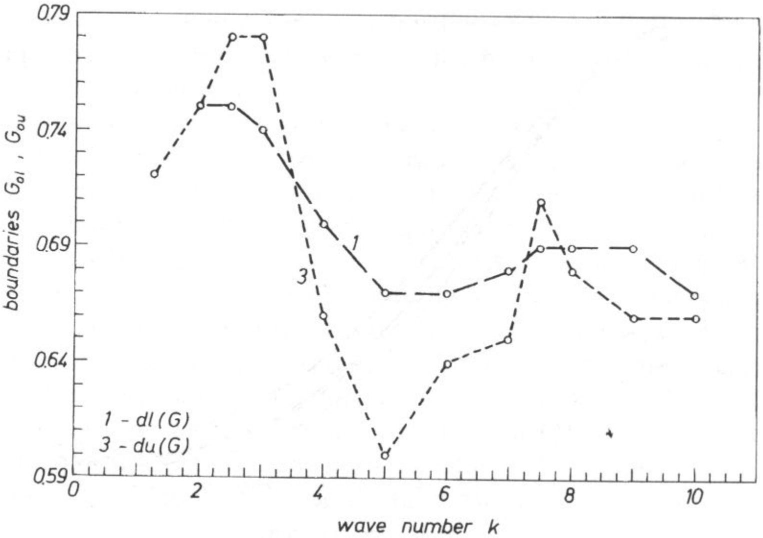

Fig. 10. Values of G_{ol} and G_{ou} as a function of wave number k; $a=0$, $\rho_c=.3$.

I.3. The position of vibrating surface in the baffle a ($b=a+1$) was changed where $\rho_c=3$, $k=2.5$ were asssumed. Due to the axisymmetry of the problem, the assumption of $a>0$ changes the circular source into a ring-shaped one. For values $a=\{0.5, 1.0\}$ the directivity functions, the same as these appearing in **I.2**, are presented in Fig. 11A, B respectively. In Fig. 11 it can easily be seen that for both values of a directivity functions of M_{ol} and M_{ou} are somewhat better than that of M_r.

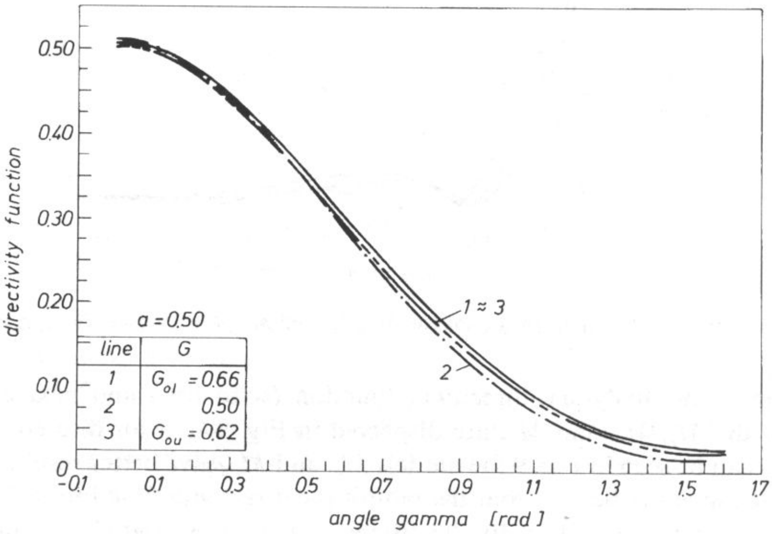

Fig. 11A. Directivity functions of the source, M_{ol}, M_r and M_{ou}; $a=.5$, $\rho_c=.3$, $k=2.5$.

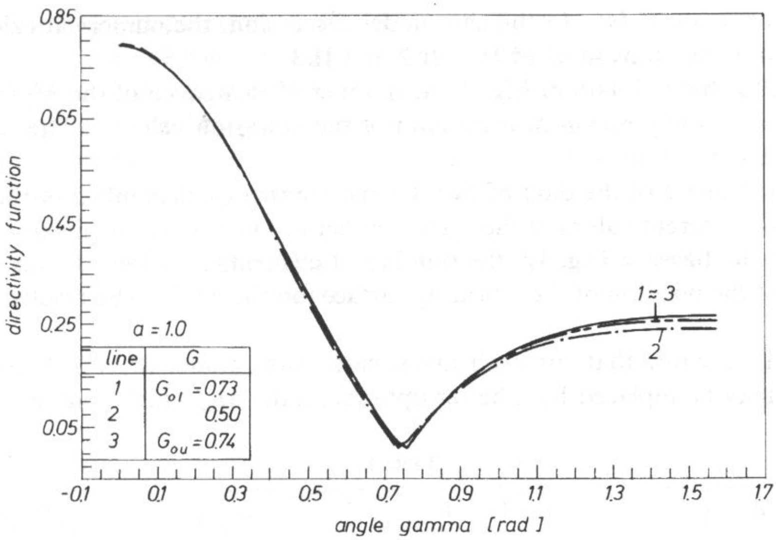

Fig. 11B. Directivity functions of the source, M_{ol}, M_r and M_{ou}; $a=1$, $\rho_c=.3$, $k=2.5$.

In Fig. 12, the discrete values of G_{ol} and G_{ou} as a function of "a" were presented. The variation of G_{ol} is more regular than that of G_{ou}.

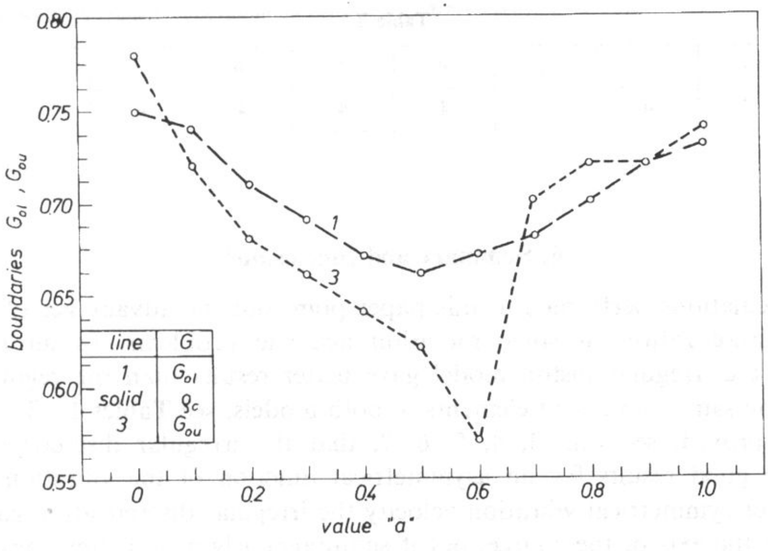

Fig. 12. Values of G_{ol} and G_{ou} as a function of a; $\rho_c=.3$, $k=2.5$.

The aim of the second part of calculations is to show that for the same number of elements M_{ol} gives better results than M_r. It was proved by giving the number of elements J of the model M_r whose directivity function is more similar to exact one than the directivity of M_{ol}. In the case under discussion, the numerical calculations were done in three steps marked **II.1**, **II.2** and **II.3**.

II.1. On the base of data of Fig. 8, the number of elements J of the M_r was found as a function of the position of maximum of the vibration velocity. The results are given in Table 1.

II.2. On the base of the data of Fig. 10, the number of elements J of the M_r was found for the different values of the wave number k. The results are given in Table 2.

II.3. On the basis of Fig. 12, the number of elements J of the M_r was found as a function of the position of the vibrating surface ion the baffle. The results are given in Table 3.

Tables 1−3 show that for each investigated case model M_r which consists of J elements may be replaced by a better optimal model M_{ol} which contains less than J elements.

Table 1

ρ_c	.2	.3	.4	.5	.6	.7	.8
J	3	4	3	3	3	4	3

Table 2

k	2	3	4	5	6	7	8
J	4	4	3	3	3	3	3

Table 3

a	.2	.3	.4	.5	.6	.7	.8
J	4	4	4	4	4	4	4

6. Summary and conclusions

The calculations performed in this paper point out the advantages of applying irregular discretization to solve radiation acoustic problems. In all test cases considered the irregular piston model gave better results than the regular model assuming the same number of elements in both models, see Tables 1−3.

It was proved, see Fig. 3, 4, 5, 6, 7, that the irregular discretization gave particularly good results for an asymmetrical function of the vibration velocity. However, for symmetrical vibration velocity the irregular discretization gave worse results near the axis of the source, but it simultaneously gave better ones near the baffle, see Fig. 5. Thus the irregular discretization ought to be applied to the source with asymmetrical vibration velocity.

The main conclusion is that the irregular discretization ought to be applied to discretization of a source with arbitrary geometry and constant acoustic variables as well as to source with arbitrary geometry and arbitrary acoustic variables.

In the paper it was pointed out, see Fig. 8, 10, 12, that the least squares distance was a better measure of convergence/divergence between directivity functions of the model and exact one than the uniform distance. Optimalization of both measures gave quite similar optimal models. However, dependently on the vibration velocity or position of the vibrating surface in the baffle and the vibration frequency, the least squares distance which assured the optimal model was more regular than the uniform distance.

Other problem of irregular discretization of the plane sources are being investigated now. The results will be published.

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