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APPLICATION OF THE ACOUSTOELASTIC EFFECT IN MEASUREMENTS OF RESIDUAL STRESSES

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This paper contains a review of non-destructive methods of residual stress measurements and results of research on the properties of the acoustoelastic effect with regard to its application in investigations of stress fields in technological materials. Values of acoustoelastic constants and temperature coefficients of velocity change were determined in several construction metals for various types of waves. Relations between wave velocity and texture, influence of texture in ultrasonic measurements of stress are presented. Ultrasonic measurement methods of absolute values of stress in screws, railway rails and railway wheels were proposed. Constructed ultrasonic meter of residual stress are described and results of comparative X-ray, ultrasonic and destructive tests are given.

Dokonano przeglądu nieniszczących metod pomiaru naprężeń własnych i opisano wyniki badania własności zjawiska elektroakustycznego pod kątem jego zastosowań w badaniach pól naprężeń w materiałach technicznych. Opisano aparaturę do precyzyjnych pomiarów prędkości fal ultradźwiękowych. Wyznaczono wartości stałych elastoakustycznych i temperaturowych współczynników zmian prędkości w kilku metalach konstrukcyjnych dla różnych typów fal. Przedstawiono związki między prędkością fal i teksturą oraz rolę tekstury w ultradźwiękowych pomiarach naprężeń. Zaproponowano ultradźwiękowe metody pomiaru bezwzględnej wartości naprężeń w śrubach, szynach kolejowych i kołach kolejowych, opisano zbudowane ultradźwiękowe mierniki naprężeń własnych i pokazano wyniki porównawczych badań rentgenowskich, ultradźwiękowych i niszczących.

Notations

a	constant dependent on screw's geometry
A	constant of acoustic anisotropy of material, birefringence,
A_0	constant of anisotropy in state without stress,
$A\sigma$	constant of stress anisotropy,
E	longitudinal modulus of elasticity,
G	shear modulus,

$K = \lambda + 2/3\mu$	bulk modulus,
l	length, third-order elasticity constant,
L	notation of longitudinal waves,
m, n	third-order elasticity constant,
N	sending head,
0	index referring to the initial state (without stress),
O	receiving head,
P	force, hydrostatic pressure,
r	radius,
R	notation of surface waves,
SH	notation of transverse waves polarized parallel to surface,
t	time,
Δt	time increment,
t^w	time of flight in the standard material,
V_{ijk}	phase velocity of waves propagating in direction i, polarized in direction j, under stress from direction k,
ΔV	velocity increment,
w_{ijk}	coefficients determining texture,
α	angle of incidence,
β_{ijk}	elastoacoustic constant of material for waves propagating in direction i, polarized in direction j, under stress from direction k,
λ	wave length,
λ, μ	Lamé elasticity constants (second-order elasticity constants),
ν	Poisson's ratio,
ϱ	mass density,
δ	stress.

1. Introduction

Residual stresses can occur in elasto-plastic materials in a homogeneous temperature field without external forces. They are formed as a result of structural changes related to mass density changes or when the yield point is exceeded in a partial volume of the material due to mechanical or thermal load. Residual stresses and stresses originating from external loads sum up and frequently have a decisive influence on the mechanical state of material. The strength of construction elements can be adequatly higher or lower, depending on the signs of residual and external stresses. Compressive residual stresses in tension under external loads areas are advantageous.

A quantitative description of residual stresses is essential in studies of strength of material and in the evaluation of exploitation properties of construction elements and machine parts. However, complicated, arduous and expensive destructive measurement methods of these stresses have hindered research work and limited investigations to the most dangerous cases, or cases causing failures most frequently.

The following methods are considered classical methods of measuring stresses: resistance tensommetry, X-ray diffractomme try, photo-elasticity and mentioned destructive methods based on the measurement of deformations during stress release. Among these, only photoelasticity is a direct method. Other methods are based on the linear dependence between measured deformation and determined stress.

Measurement procedures and apparatus for resistance tensommetry have been well developed. However, it is limited by a lack of possibility of measuring absolute values of stress. Only stress increments can be measured with the application of this technique, and solely on the surface of tested object. Photo-elasticity can be used in investigations of stress distributions, but in models made from transparent materials only. With the X-ray technique stresses can be measured only on the surface. Numerous phenomena, influenced by stress, are experimentally applied in stress measurements. The following are most important:

neutron diffraction,
Berkhausen's magnetic noise,
acoustic emission accompanying magnetizing of ferromagnetic materials,
velocity changes of ultrasonic waves during magnetization changes,
microwave absorbtion,
damping of ultrasonic waves,
temperature dependence of velocity of ultrasonic waves,
acoustoelastic effect.

Results of stress measurements done with the application of mentioned phenomena usually are not solely stress dependent, but also depend on other factors, including parameters of the material such as: chemical composition, structure, thermal and mechanical processing, history of the sample. Only investigations of materials with high degree of homogeneity provide reliable quantitative results. Mechanical construction materials rarely meet this requirement.

This paper is concerned with research on the utilization of the relationship between velocity of ultrasonic waves and stress (acoustoelastic effect), which seems promising in practical applications in measurements of residual stress on the surface as well as inside technological materials.

2. Acoustoelastic effect

Velocity of acoustic waves in a solid body depends on forces of atom-atom interactions and mass of atoms transmitting wave motion. The linear theory of elasticity leads to known expressions of velocity of longitudinal and transverse waves in an unlimited isotropic solid:

$$V_{11}=\sqrt{\frac{\lambda+2\mu}{\varrho}}=\sqrt{\frac{E}{\varrho}\,\frac{1-\nu}{(1+\nu)(1-2\nu)}}\tag{1}$$

$$V_{12}=\sqrt{\frac{\mu}{\varrho}}=\sqrt{\frac{E}{2\varrho}\,\frac{1}{1+\nu}}=\sqrt{\frac{G}{\varrho}}$$

In these expressions V is the phase velocity of the wave, first index determines the direction of wave propagation in a cartesian coordinates system with axises 1, 2 and 3, second index determines the direction of particles vibrations in the wave, ϱ — mass density, E — and G — shear and elastic moduli, λ — and μ — Lame's constants (second-order elasticity constants) ν — Poisson's ratio.

The classical theory of elasticity assumes infinitesimal deformations of perfectly elastic bodies and does not provide for a velocity-stress dependence. When nonlinear properties of solids are included, the velocity-stress dependence is achieved. In a paper published in 1953 [1] Hughes and Kelly derivated formulae for velocity of elastic waves in solids subjected to stress (infinitesimal deformations superimposed on finite deformation). They took advantage of Murnaghan's theory of finite deformations, including third-order terms in the expression for elastic energy of a deformed body. Achieved by these authors dependencies between velocity of low amplitude elastic waves and stress are considered fundamental in the description of the elastoacoustic phenomenon:

$$\varrho_0V_{111}^2=\lambda+2\mu-(\sigma/3K_0)\left(\frac{\lambda+\mu}{\mu}(4\lambda+10\mu+4m)+\lambda+21\right),\tag{2}$$

$$\varrho_0V_{113}^2=\lambda+2\mu-(\sigma/3K_0)\left(\frac{2\lambda}{\mu}(\lambda+2\mu+m)-21\right),\tag{3}$$

$$\varrho_0V_{131}^2=\mu-(\sigma/3K_0)\left(4\lambda+4\mu+m+\frac{\lambda n}{4\mu}\right),\tag{4}$$

$$\varrho_0V_{133}^2=\mu+(\sigma/3K_0)\left(\lambda+2\mu+m+\frac{\lambda n}{4\mu}\right),\tag{5}$$

$$\varrho_0V_{132}^2=\mu+(\sigma/3K_0)\left(2\lambda-m+\frac{n}{2}+\frac{\lambda n}{\mu}\right),\tag{6}$$

Similarly, for hydrostatic pressure

$$\varrho_0V_{11P}^2=\lambda+2\mu-(P/3K_0)(61+4m+7\lambda+10\mu),\tag{7}$$

$$\varrho_0V_{13P}^2=\mu-(P/3K_0)\left(3m-\frac{n}{2}+3\lambda+6\mu\right),\tag{8}$$

where ϱ_0 – mass density of not deformed medium; v – wave velocity in deformed material; λ and μ – Lamé's elasticity constants; m, n, l – Murnaghan's third-order elasticity constants, σ – uniaxial stress; $K_0 = \lambda + 2/3\mu$ – bulk modulus of not deformed medium, P – pressure. The third index of velocity denotes the direction of stress. Expressions (2) to (8) are used to determine values of third-order elasticity constants on the basis of wave velocity changes due to given stress increments. Including expressions (1) and (2) for velocity of acoustic waves in a not deformed isotropic body, and accepting that velocity increments, $v - v_0$, due to stress are small in comparison to absolute values of velocity, v and v_0 (what leads to an approximation: $V + V_0 \cong 2V_0$), we achieve relationships between relative changes of velocity and stress from equations (2) to (8). Such a relationship for longitudinal waves propagating in the direction of stress is as follows:

$$\left(\frac{V-V_0}{V_0}\right)_{111} = -\frac{\dfrac{\lambda+\mu}{\mu}(4\lambda+10\mu+4m)+\lambda+21}{2(\lambda+2\mu)(3\lambda+2\mu)}\cdot\sigma_1. \tag{9}$$

Factors of proportionality between stress and relative changes of wave velocity are called acoustoelastic coefficients. These coefficients are expressed by second- (λ, μ) and third-order (l, m, n) elasticity constants. Linearized relationships, like expression (9), ore the fundamental relationships of ultrasonic tensommetry. They have the following form:

$$\begin{gathered} V_{ij} = V_{ij}^0(1+\beta_{ijk}\sigma_k) \\ t_{ij} = t_{ij}^0/(1+\beta_{ijk}\sigma_k) \end{gathered} \tag{10}$$

where t and t_0 are travel times over a determined path in a material under stress and material without stress (material in neutral state), respectively, while β is a acoustoelastic coefficient.

Experiments confirm the linearity of the dependence between stress and relative change of wave velocity. Fig. 1 presents increments of travel times of longitudinal L, transverse T and surface R waves propagating in the direction of stress in a St3 steel sample, due to stress. Changes of travel times of waves were measured on a length between receiving heads O_1 and O_2, equal to 196 mm for longitudinal waves, 107 mm for transverse and surface waves.

Table 1 contains values of acoustoelastic constants for several structural metals. The amount of velocity changes due to stress depends on the sort of material, direction of wave propagation and direction of vibration of particles in the wave (direction of polarization) with respect to direction of stress. Longitudinal waves propagating along the direction of stress are most sensitive to stress. The velocity increment of longitudinal waves propagating in the direction of stress in a sample of carbon steel, resulting from a stress change of 10 MPa, is equal to approximately 0.75 m/s. This makes about 0.013 % of the value of velocity of longitudinal waves in steel. The corresponding velocity increment for transverse waves is equal to about

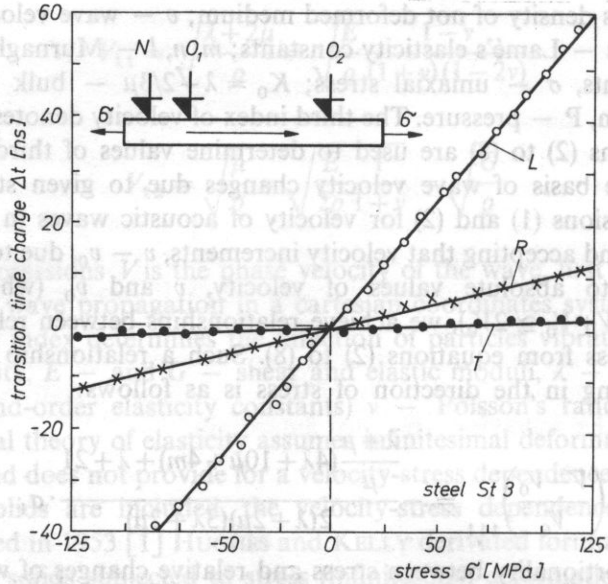

FIG. 1. Changes, Δt, of travel time of longitudinal waves L, transverse waves T, surface waves R, propagating in the direction of stress versus stress δ

0.05 m/s, and for surface waves – about 0.15 m/s. In a state of compound stress the effect of individual stress components on wave velocity sums up. From equations (3), (5) and (6) for waves propagating in the direction of thickness of a plate under biaxial state of stress we achieve the following relationships:

$$\frac{V_{33}-V_{33}^0}{V_{33}^0} = \beta_{331}(\sigma_2+\sigma_3), \tag{11}$$

$$\frac{V_{31}-V_{32}}{\frac{1}{2}(V_{31}^0+V_{32}^0)} = (\beta_{311}-\beta_{321})(\sigma_1-\sigma_2), \tag{12}$$

$$\frac{V_{31}+V_{32}-V\frac{1}{2}(V_{31}^0+V_{32}^0)}{V_{31}^0+V_{32}^0} = (\beta_{311}+\beta_{322})(\sigma_2+\sigma_3). \tag{13}$$

In these formulae V^0 denotes wave velocity for zero stress. Naturally, $V_{31}^0 = V_{32}^0$ for an isotropic material.

Expression (12) describes acoustic birefrigence. Due to stress an isotropic medium becomes anisotropic. The difference of velocity of transverse waves propagating in the direction of plate's thickness and polarized in directions of principal stresses is the measure of anisotropy due to stress.

Diagrams in Fig. 2 present times of flight of longitudinal, t_{33}, and transverse, t_{32},

Table 1. Acoustoelastic constants of aluminium, copper and steel. Consecutive indices of the symbol of the constant denote directions of propagation of a wave beam, polarization of vibrations of particles in the wave and direction of stress

Material	Elastoacoustic constant 10^{-5}MPa^{-1}				
	β_{111}	β_{221}	β_{121}	β_{211}	β_{231}
Aluminium	−7.75	+1.13	−2.19	−4	+0.89
Copper	−1.88	−0.18	0	−1.07	+0.2
Armco iron	−2.69	+0.49	+0.13	−0.48	−3.13
Nickel steel 535	−1.13	+0.06	−0.05	−0.66	+0.03
Steel 60C2H2A	−0.99	+0.14	−0.0	−0.70	+0.17
Steel Hecla 37	−1.39	+0.02	−0.12	−0.73	+0.03
Steel M56	−2.06	+0.15	−0.54	−1.17	+0.47

waves in terms of a force compressing a disc from PA6N alloy. The time of sixteen transitions of a pulse through the thickness of the disc was measured for longitudinal waves and of eight transitions for transverse waves. In effect of a point pressure P, acting along the diameter of the disc, a state of biaxial stress is created in the centre of the disc, where compressive stress is $\delta_1 = -2P/\pi gd$ and tensile stress is $\delta_2 = 6P/\pi gd$ (d is the diameter and g is the thickness of the disc). Linear

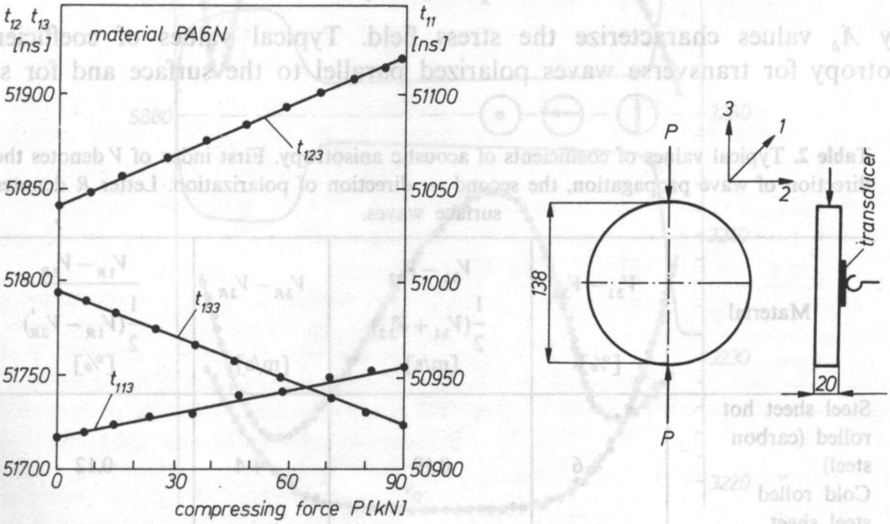

FIG. 2. Changes of travel time of longitudinal waves t_{33}, and transverse waves, t_{31} and t_{32}, versus force P compressing a disc from PA6N alloy

dependencies between stress and relative velocity change can be applied directly in measurements of stress increments. Adequate electroacoustic constants are determined in tensile tests. Because of a significant temperature dependence of the velocity of ultrasonic waves it is necessary to include adequate corrections for temperature in ultrasonic measurements of stress. Fortunately velocity changes are in linear dependence with temperature changes. Technological materials usually exhibit considerable heterogeneity and anisotropy of acoustic properties. In various areas of material free from stress the value of velocity of ultrasonic waves, V_0, can be different. The limits of heterogeneity of acoustic properties of technological metals are not known precisely. Results of wave velocity measurements performed on annealed samples cut out from various areas of the same element indicate the scope of heterogeneity. In the case of NC6 tool steel the greatest velocity difference of longitudinal waves was equal to 3.2 m/s, for surface waves – 5.2 m/s and for transverse waves – 5.4 m/s. In a carbon steel sheet local velocity differences amounted to: 10.8 m/s for longitudinal waves, 6.9 m/s for surface waves and 7.2 m/s for transverse waves. Such velocity differences can be incorrectly attributed to local differences of residual stress. The V_0 value in the area of material under investigation has to be known in order to determine the absolute value of stress. When measurements of acoustic anisotropy are applied in investigations of stress fields, then the anisotropy of the material free from stress has to be considered. Anisotropy in a stress free state in policrystalline materials is related to the preferential orientation of grains (texture) or to the directional arrangement of impurities. The measured coefficient of anisotropy, A, is a sum of anisotropy in a stress free state, A_0, and anisotropy caused by stresses, A_δ,

$$A = A_0 + A_\delta. \tag{14}$$

Only A_δ values characterize the stress field. Typical values of coefficients of anisotropy for transverse waves polarized parallel to the surface and for surface

Table 2. Typical values of coefficients of acoustic anisotropy. First index of V denotes the direction of wave propagation, the second – direction of polarization. Letter R denotes surface waves.

Material	$V_{31}-V_{32}$ [%]	$\dfrac{V_{31}-V_{32}}{\frac{1}{2}(V_{31}+V_{32})}$ [m/s]	$V_{3R}-V_{2R}$ [m/s]	$\dfrac{V_{1R}-V_{2R}}{\frac{1}{2}(V_{1R}-V_{2R})}$ [%]
Steel sheet hot rolled (carbon steel)	+6	0.19	+4	0.12
Cold rolled steel sheet (carbon steel)	+280	8.60	+122	3.90
Sheet aluminium	−2.5	0.08	−150	0.05

Table 3. Velocity of ultrasonic waves in a sample of austenitic steel 23 Cr, 12 Ni

Wave	V[m/s]	Wave	V[m/s]	Wave	V[m/s]
V_{33}	5369	V_{32}	3852	V_{23}	3847
V_{22}	5775	V_{31}	3853	V_{12}	2950
V_{11}	5812	V_{21}	2960	V_{13}	3862

waves are given in Table 2. Direction 1 is the direction of rolling, direction 2 is a direction perpendicular to the rolling direction in the plane of rolling.

Table 3 presents velocity values of bulk waves propagating in various directions in a sample cut from austenitic steel. The velocity distribution of ultrasonic waves in the cross section of a railway rail is differentiated. In Fig. 3 velocity changes of longitudinal and transverse waves propagating along the length of the rail versus distance from the rail surface are shown. The velocity distribution reflects texture changes of steel formed in the rolling process. Anisotropy has to be taken into consideration in investigations of stress fields in materials exhibiting anisotropy in

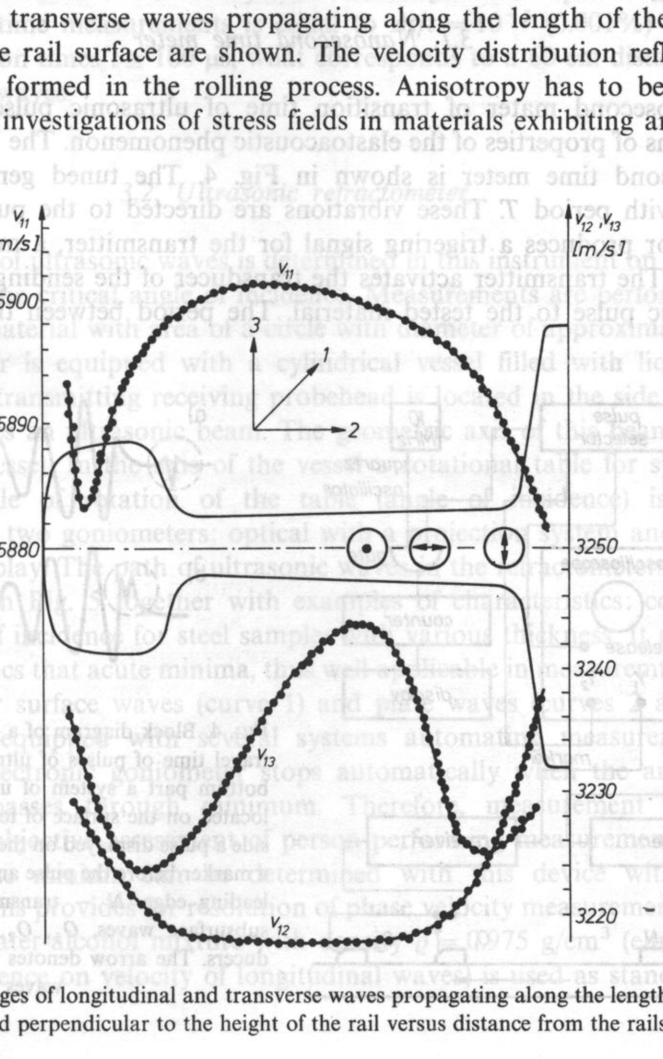

FIG. 3. Velocity changes of longitudinal and transverse waves propagating along the length of the rail and polarized parallel and perpendicular to the height of the rail versus distance from the rails rolling surface

a state free from stress. In certain cases the value of wave velocity, V_0, as well as the coefficient of anisotropy, A_0, in a state free from stress can be measured in an area under test before stress is applied. However, frequently such data is inaccessible and in order to determine stress methods of determining V_0 have to be applied, which are based for example, on measurements of velocities of many types of wares, with velocities to a different degree dependent on stress, or take advantage of such combinations of wave velocities, which are independent of texture.

Several possibilities will be discussed with descriptions of ultrasonic measurements of stresses in technological materials.

3. Measuring technique of velocity of ultrasonic waves

3.1. *Nanosecond time meter*

A nanosecond mater of transition time of ultrasonic pulses was built for investigations of properties of the elastoacoustic phenomenon. The block diagram of the nanosecond time meter is shown in Fig. 4. The tuned generator produces vibrations with period T. These vibrations are directed to the pulse selector. The pulse selector produces a trigering signal for the transmitter, a marker signal and gate signal. The transmitter activates the transducer of the sending head and sends an ultrasonic pulse to the tested material. The period between the production of

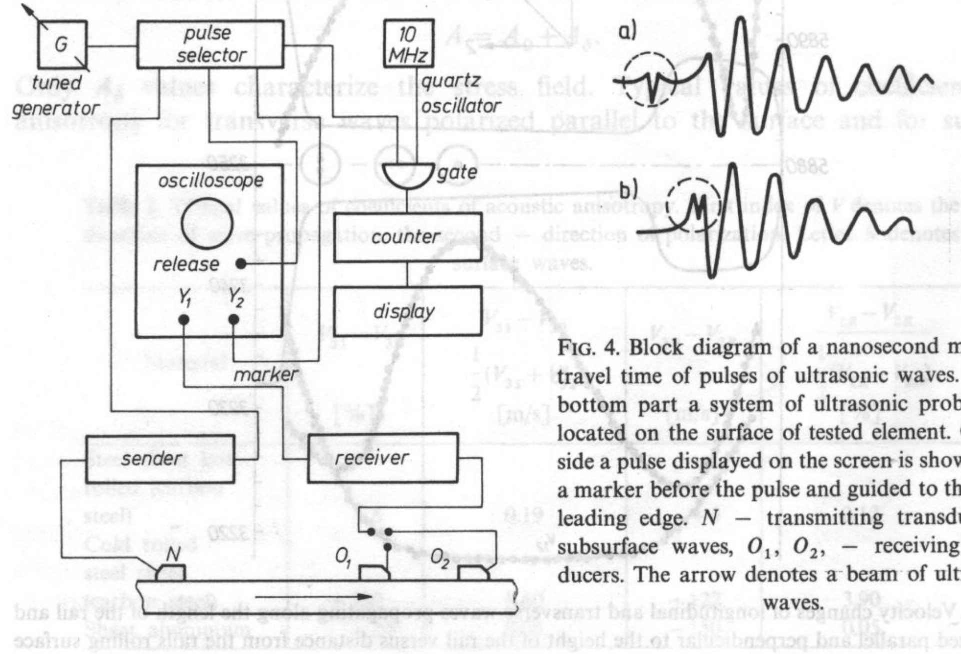

FIG. 4. Block diagram of a nanosecond meter of travel time of pulses of ultrasonic waves. In the bottom part a system of ultrasonic probeheads located on the surface of tested element. On the side a pulse displayed on the screen is shown with a marker before the pulse and guided to the pulse leading edge. N — transmitting transducer of subsurface waves, O_1, O_2, — receiving transducers. The arrow denotes a beam of ultrasonic waves.

successive ultrasonic pulses (repetition time) is equal to 100 *T*. After a period of time equal to 9 T the time base of the oscilloscope is released. A time marker, appearing after time period 10 *T* from the release of the transmitter, is superimposed on the received signal, presented on the screen of the oscilloscope tube. By means of fine tuning of generator *G*, its vibration period *T*, can be so adjusted, that the time marker is located in a chosen place on the display of received ultrasonic pulse. In this case the transition time of ultrasonic waves will be equal to 10 *T*. The time measurement with ±1 ns accuracy is achieved by a pulse count in time equal to 10^4 *T* from a standard quartz oscillator with 10 MHz frequency and rounding of the indicator of of last counter. Repeatability of successive measurements of time of flight equal to ±1 ns, for wave frequency 4 MHz, was achieved by using a *V*-shaped marker (Fig. 4) and by applying the criterion of equal arms in the process of superimposing the marker on the maximum. The accuracy of electronic systems is equal to ±0.1 ns. The relative error of time measurements is equal to $\Delta t/t = 10^{-5}$ (0.001%) for average measured transition times $t \cong 100$ μs, what corresponds to a 60 cm distance in steel for longitudinal waves.

3.2. *Ultrasonic refractometer*

The velocity of ultrasonic waves is determined in this instrument on the basis of measurement of the critical angle of incidence. Measurements are performed in the surface layer of material with area of a circle with diameter of approximately 5 mm. The refractometer is equipped with a cylindrical vessel filled with liquid during measurement. A transmitting receiving probehead is located in the side wall of the vessel. It generates an ultrasonic beam. The geometric axis of this beam intercepts the axis of the vessel. In the axis of the vessel a rotational table for specimens is placed. The angle of rotation of the table (angle of incidence) is measured independently by two goniometers: optical with a projection system and electronic with a digital display. The path of ultrasonic waves in the refractometers measuring vessel is shown in Fig. 5 together with examples of characteristics: coefficient of reflection-angle of incidence for steel samples with various thickness. It results from these characteristics that acute minima, thus well applicable in measuremts, occur for critical angles for surface waves (curve 1) and plate waves (curves 2 and 3). The refractometer is equipped with several systems automating measurements. The counter of the electronic goniometer stops automatically when the amplitude of reflected pulse passes through minimum. Therefore, measurement results are independent of subjective assessment of person performing measurements. Angular positions of these minima can be determined with this device with accuracy exceeding ±1′. This provides for resolution of phase velocity measurements of about 0.01% when a water-alcohol mixture with density $\varrho = 0.975$ g/cm^3 (elimination of temperature influence on velocity of longitudinal waves) is used as standard liquid.

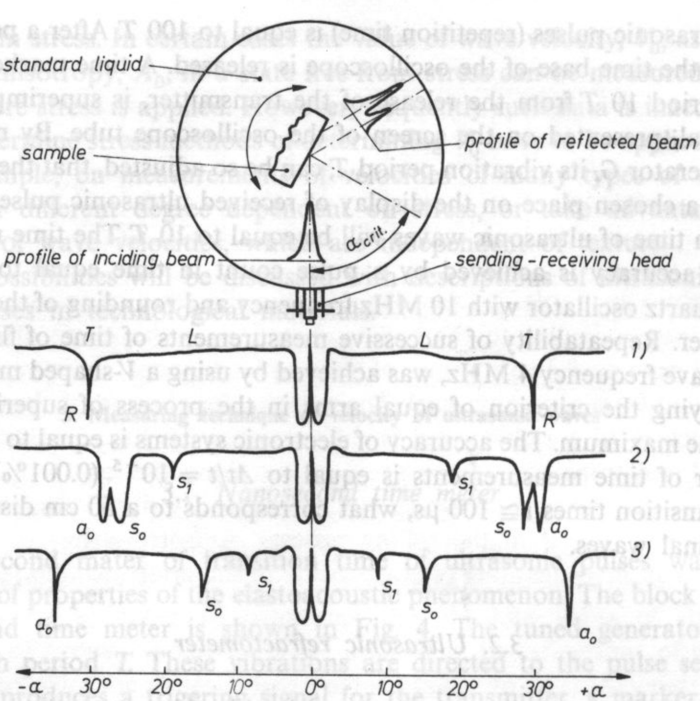

FIG. 5. Principle of measurement of critical angles of incidence on the liquid-solid interface with an ultrasonic refractometer and examples of dependencies of the coefficient of reflection on the angle of incidence α. Curves are for steel samples with thickness 20; 0.5; 0.3 mm respectively. Letters L, T and R denote longitudinal transverse and surface waves, respectively, and critical angles for these waves. a_0, s_0 and s_1 – minima corresponding to successive modes of plate waves

4. Examples of ultrasonic measurements of residual stresses

4.1. Stress measurements in bolts

Stress increment, $\Delta\delta$, in a bolt can be determined from measurements of changes of travel times of ultrasonic waves along the axis of the bolt. If the travel time is measured before ($\delta = 0$) and after ($\delta = 0$) screwing tight the bolt, then the relative change of travel time of waves along the bolt's axis are directly proportional to stress and relationships $\Delta t = f(\Delta\delta)$, determined from experiment for bolted joints in a given construction, can be applied in measurements of stress changes due to screwing.

Fig. 6 presents results of measurements of relative changes of travel times of longitudinal and transverse waves propagating along the length of a screw made of 25H2MF steel during tension [2]. The measured value of relative change of travel time of ultrasonic waves due to path increase of wave resulting from elastic deformation, and to velocity change caused by stress influence on velocity is:

$$\frac{t_0 - t}{t} = \frac{\Delta t}{t_0} = \left(\frac{1}{E} + \beta\right)\sigma \tag{15}$$

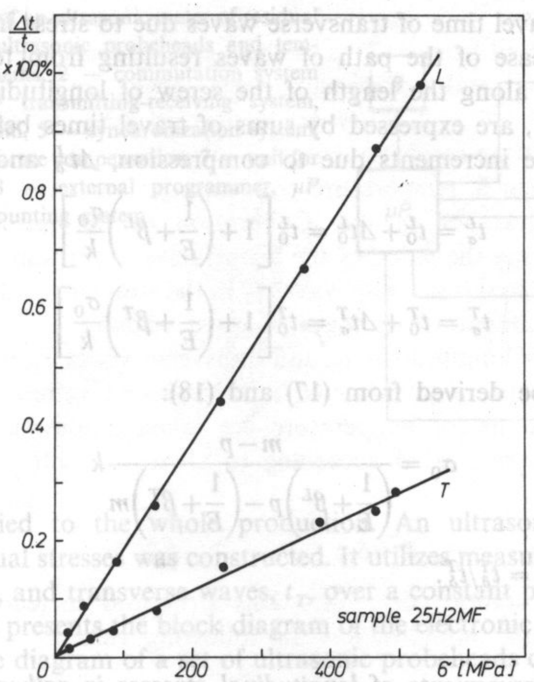

FIG. 6. Relative increments of travel times of longitudinal L and transverse T waves propagating in the direction of stress versus tensile stress. Sample from 25H2MP steel.

where E is the longitudinal modulus and β is an elastoacoustic constant. The relationship is in force when ultrasonic waves propagate along a path in a medium with stress δ. In bolts stresses are distributed along the hickness of the screw and the recorded value $\Delta t/t_0$ is the measure of mean stress, δ_{mid}, along the path of the wave. Stress in the smooth segment of the bolt is

$$\delta_0 = k \cdot \delta_{\mathrm{mid}}. \tag{16}$$

Naturally, coefficient k exceeds unity. As a result of a gross approximation $k = l/(l_0 + a)$, where l is the total length of bolt, l_0 – length of the smooth part with uniform stress, and a – constant dependent on construction parameters of the bolt, such as height off bolt-cap, length of threaded part, parameters of the thread, etc.

The value of coefficient k, dependent on the shape of the bolt and design of bolted joint, can be determined from experiment, as well as from calculations [2]. If we have previously measured the value of travel time, t_0, before screwing the bolt tight, then stress in the bolt can be calculated from expression (15). When the value t_0 remains unknown, then stress in a bolt can be determined on the basis of measurements of times of flight along the length of the bolt of longitudinal as well as transverse waves. The idea of the measurement is based on the effect that longitudinal waves are many times more sensitive to stress than transverse waves.

The increment of travel time of transverse waves due to stress in the bolt is caused mainly by the increase of the path of waves resulting from tension of the bolt.

Times of flight along the length of the screw of longitudinal waves, t^L_δ, and transverse waves, t^T_δ, are expressed by sums of travel times before screwing tight, t^L_0 and t^T_0, and time increments due to compression, Δt^L_δ and Δt^T_δ:

$$t^L_\sigma = t^L_0 + \Delta t^L_0 = t^L_0\left[1+\left(\frac{1}{E}+\beta^L\right)\frac{\sigma_0}{k}\right] \tag{17}$$

$$t^T_\sigma = t^T_0 + \Delta t^T_\sigma = t^T_0\left[1+\left(\frac{1}{E}+\beta^T\right)\frac{\sigma_0}{k}\right] \tag{18}$$

The δ_0 value can be derived from (17) and (18):

$$\sigma_0 = \frac{m-p}{\left(\frac{1}{E}+\beta^L\right)p-\left(\frac{1}{E}+\beta^T\right)m}k \tag{19}$$

where: $m = t^T_0/t^L_0$, $p = t^T_\delta/t^T_\delta$.

4.2. *Measurements of longitudinal stresses in railway rails*

Plastic deformations in the process of final straightening of rails are the main source of longitudinal residual stresses in railway rails. A typical distribution of residual stresses in the cross section of a rail after straightening is shown in Fig. 7. Too high tensile stress in the rail head and base promote cracking of rails along the web, while tensile stresses in the rail base sum up with exploitation stresses increasing the danger of fatigue cracking. Previously destructive methods were applied in assessments of the level of residual stresses in rails. These methods are expensive and

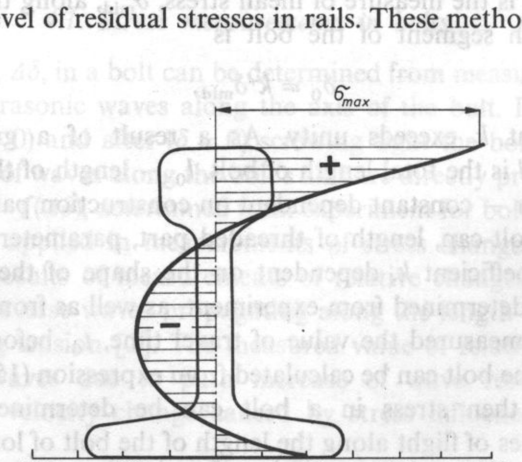

FIG. 7. Typical distribution of average longitudinal residual stresses in a cross-section of a straightened rail

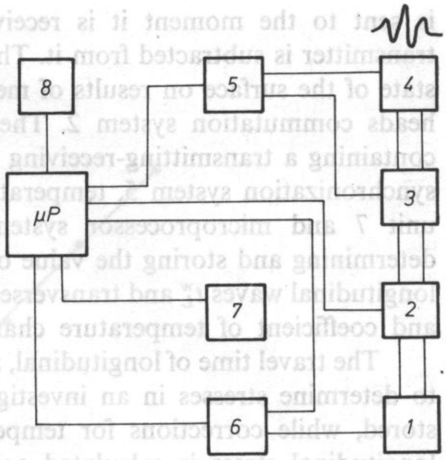

FIG. 8. Block diagram of an ultrasonic meter of residual stresses. 1 – set of ultrasonic probeheads and temperature-sensitive elements, 2 – commutation system for probeheads, 3 – transmitting-receiving system, 4 – visualization system, 5 – synchronization system, 6 – system for temperature compensation, 7 – unit for time measurements, 8 – external programmer, μP – counting system

can not be applied to the whole production. An ultrasonic meter measuring longitudinal residual stresses was constructed. It utilizes measurements of travel time of longitudinal, t_L, and transverse waves, t_T, over a constant path in the material of the rail [3]. Fig. 8 presents the block diagram of the electronic part of the meter and Fig. 9 presents the diagram of a set of ultrasonic probeheads collaborating with the meter.

The transmitting transducer N_L generated pulses of longitudinal waves, which generate in the tested material subsurface longitudinal waves (when the angle of incidence is equal to the first critical angle) or transverse waves (when the angle of incidence is equal to the second critical angle) when they are refracted passing through a plastic wedge. The transition of ultrasonic pulses is registered in turn by separate pairs of receiving transducers for longitudinal waves (O_1^L and O_2^L) and transverse waves (O_1^T and O_2^T). Measurements of travel time on the path between receiving transducers are done with the differential method, i. e., travel time is measured from the moment when the pulse is sent to the moment it is received by the further receiver and then the travel time measured from the moment when the pulse

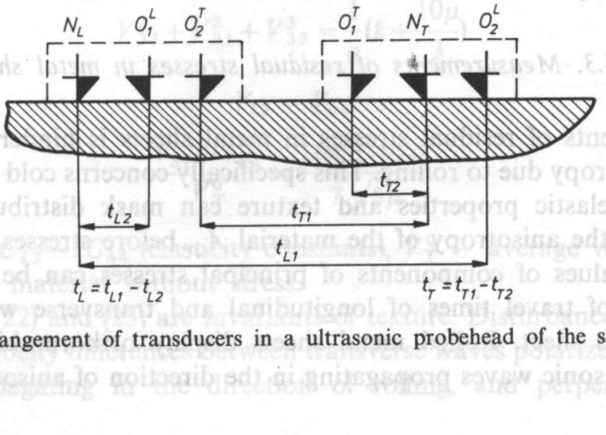

FIG. 9. Arrangement of transducers in a ultrasonic probehead of the stress meter

is sent to the moment it is received by the receiving transducer closer to the transmitter is subtracted from it. This method greatly decreases the influence of the state of the surface on results of measurements. Heads are switched in turn by the heads commutation system 2. They are connected with an electronic apparatus containing a transmitting-receiving system 3, display system of ultrasonic pulses 4, synchronization system 5, temperature compensating system 6, time measurement unit 7 and microprocessor system. Calibration of the instrument consists in determining and storing the value of the acoustoelastic constant β_L, travel times of longitudinal waves t_L^w and transverse waves t_T^w in the standard material without stress and coefficient of temperature changes of wave velocity.

The travel time of longitudinal, t_L, and transverse waves, t_T, is measured in order to determine stresses in an investigated rail. Measured values of travel times are stored, while corrections for temperature are taken automatically. The value of longitudinal stress is calculated according to algorithm (20)

$$\sigma = \frac{1}{\beta_L t_L}\left[t_L^w + \Delta t_L^s - t_L\right], \tag{20}$$

where Δt_L^s is the structural correction. This structural correction, calculated for assumed constant bulk modulus, has the following value:

$$\Delta t_L^s = \frac{4}{3}(l_T/l_L)^2\,(t_L/t_T)^3 \cdot \Delta t_T \tag{21}$$

where: l_T and l_L are paths of transverse and longitudinal ultrasonic waves, Δt_T is the difference between travel times of transverse waves on the same path in the standard material and in the material of the rail under test.

Fig. 10 presents the dependence of travel time of longitudinal waves between receiving transducers O_1 and O_2, on stress for 90PA steel. This dependence is used in the process of calibration of the stress meter. The value of the acoustoelastic constant is equal $\beta_L = -1.12 \times 10^{-5}$ MPa^{-1} for this material. The correlation coefficient is equal 0.9995. The measurement of stress in one point takes about 1 minute. This allowed full control of the level of residual stresses in rails produced by the Katowice Steel Mill.

4.3. Measurements of residual stresses in metal sheets

Measurements of residual stresses in metal sheets is hindered mainly by the materials anisotropy due to rolling. This specifically concerns cold rolled steel sheets. Differences in elastic properties and texture can mask distributions of residual stresses. When the anisotropy of the material A_0, before stresses are introduced is known, then values of components of principal stresses can be determined from measurements of travel times of longitudinal and transverse waves through the thickness of the sheet. Rolled metal sheets display orthorombic symmetry. The velocity of ultrasonic waves propagating in the direction of anisotropy is expressed

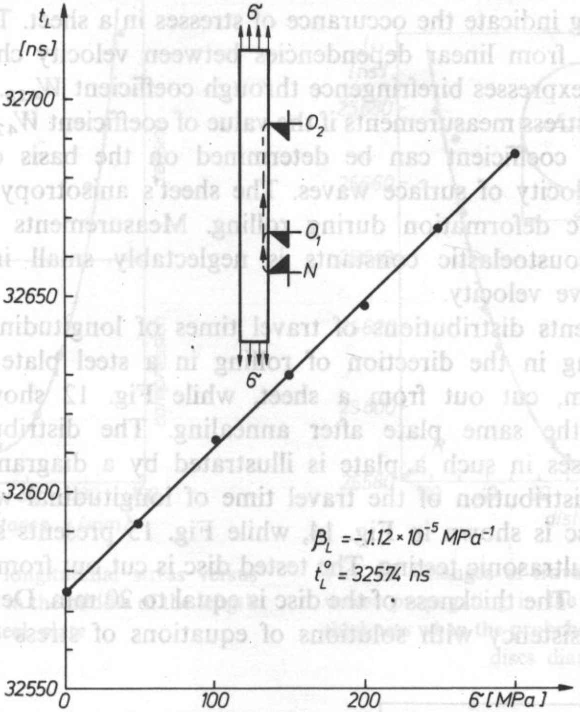

FIG. 10. Acoustoelastic characteristic of 90PA steel for longitudinal waves propagating in the direction of stress

by elasticity constants of the monocrystal and coefficients w_{400}, w_{420} and w_{440}, which determine the distribution of crystallographic orientations grain with respect to selected directions in the sample [4]. Coefficients w_{klm} are coefficients in an expansion into Legendre's multinomials of the function of distribution of orientations of crystal grains. It 1 is the direction of rolling, 2 – direction perpendicular to the direction of rolling, and 3 – direction of thickness, then the following relations occur between velocities of various wave modes in a sheet without stress:

$$V_{33}^2+V_{31}^2+V_{32}^3 = \frac{1}{\varrho}\left(k+\frac{10\mu}{3}\right) \tag{22}$$

$$V_{12} = V_{21} \tag{23}$$

$$\frac{V_{31}-V_{32}}{V_T^0} = \frac{16\pi^2 C w_{420}}{7\sqrt{5}\mu} \tag{24}$$

where $C = C_{11}-C_{12}-2C_{44}$ (elasticity constants), V_T^0 – average velocity of transverse waves in a material without stress.

Expressions (22) and (23) are invariants of texture. Disturbances of the sum of squares (22) or velocity differences between transverse waves polarized parallel to the surface and propagating in the direction of rolling, and perpendicular to the direction of rolling indicate the occurance of stresses in a sheet. The value of stress can be calculated from linear dependencies between velocity changes and stress. Relationship (24) expresses birefringence through coefficient W_{420}. This relationship can be applied in stress measurements if the value of coefficient W_{420} is known. Then, the value of this coefficient can be determined on the basis of the directional dependence of velocity of surface waves. The sheet's anisotropy changes with an increase of plastic deformation during rolling. Measurements indicate that the anisotropy of acoustoelastic constants is neglectably small in comparison to anisotropy of wave velocity.

Fig. 11 presents distributions of travel times of longitudinal and transverse waves propagating in the direction of rolling in a steel plate with dimensions 40 × 100 × 400 mm, cut out from a sheet, while Fig. 12 shows corresponding distributions in the same plate after annealing. The distribution of residual longitudinal stresses in such a plate is illustrated by a diagram in Fig. 13.

The radial distribution of the travel time of longitudinal waves through the thickness of a disc is shown in Fig. 14, while Fig. 15 presents stress components determined from ultrasonic testing. The tested disc is cut out from a rolled rod with diameter 138 mm. The thickness of the disc is equal to 20 mm. Determined relations exhibit good consistency with solutions of equations of stress equilibrium.

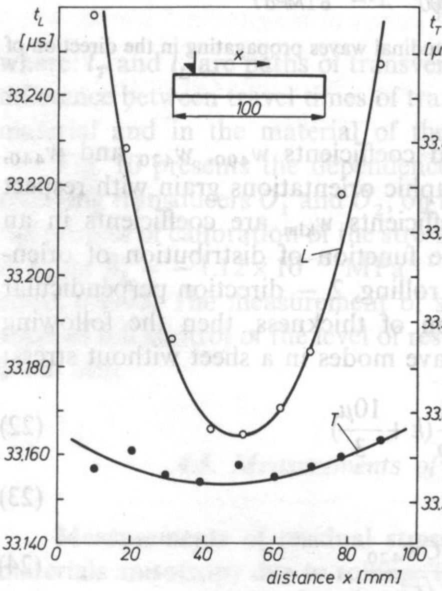

FIG. 11. Changes of travel time of subsurface longitudinal L and transverse T waves when a head is shifted on the plates surface perpendicular to the direction of rolling

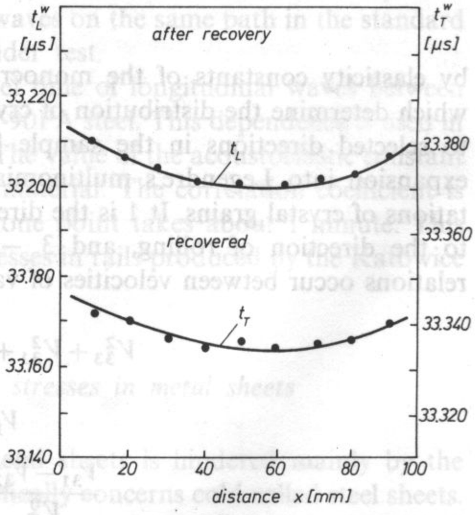

FIG. 12. Changes of travel time of longitudinal L and transverse T waves over a constant path in the direction of rolling in terms of distance from the edge after stress relieving of the plate. Data for the plate before stress relief are presented in Fig. 11

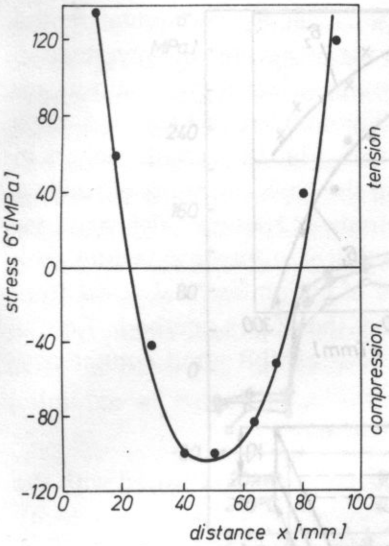

FIG. 13. Changes of longitudinal stress versus distance from the edge in the middle of the length of steel plate

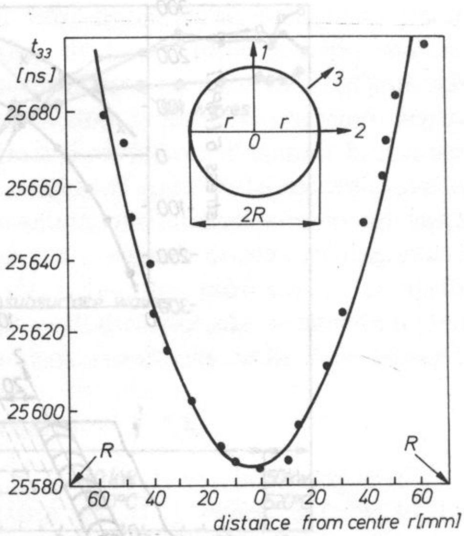

FIG. 14. Changes of travel time of longitudinal waves propagating in the direction of the discs thickness when the probehead is shifted along the discs diameter

Diagrams in Fig. 16 present distributions of stresses introduced into a steel plate due to welding, determined with ultrasonic technique [5]. Ultrasonic measurements of welding stresses can be applied in quick assesments of the degree of stress relief of welded constructions.

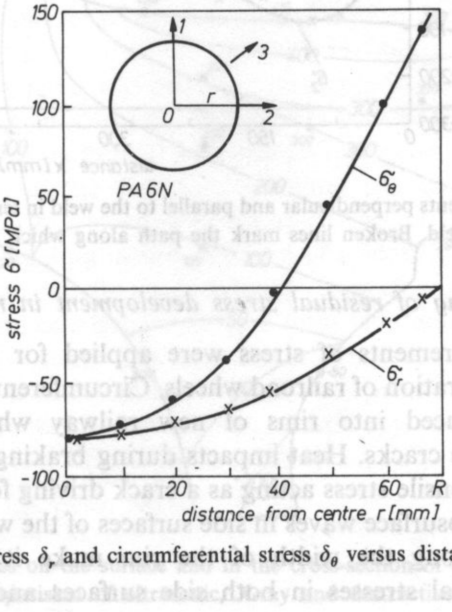

FIG. 15. Changes of radial stress δ_r and circumferential stress δ_θ versus distance from centre of the disc

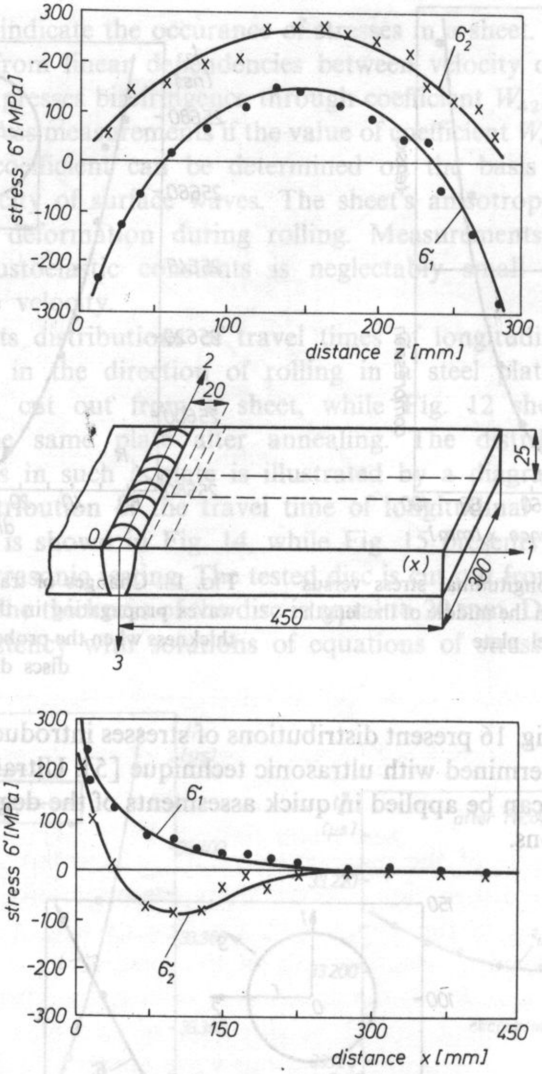

FIG. 16. Changes of components perpendicular and parallel to the weld in terms of distance along the weld and perpendicular to the weld. Broken lines mark the path along which ultrasonic heads were shifted

4.4. *Monitoring of residual stress development in railroad wheels*

Ultrasonic measurements of stress were applied for monitoring of residual stresses during the operation of railroad wheels. Circumferential compressive stresses are purposely introduced into rims of new railway wheels. They hinder the development of fatigue cracks. Heat impacts during braking can cause a change of residual stresses into tensile stress acting as a crack driving forces. At the same time, measurements with subsurface waves in side surfaces of the wheel rim and transverse waves propagating along the width of the rim make it possible to determine distributions of residual stresses in both side surfaces and radial distribution of

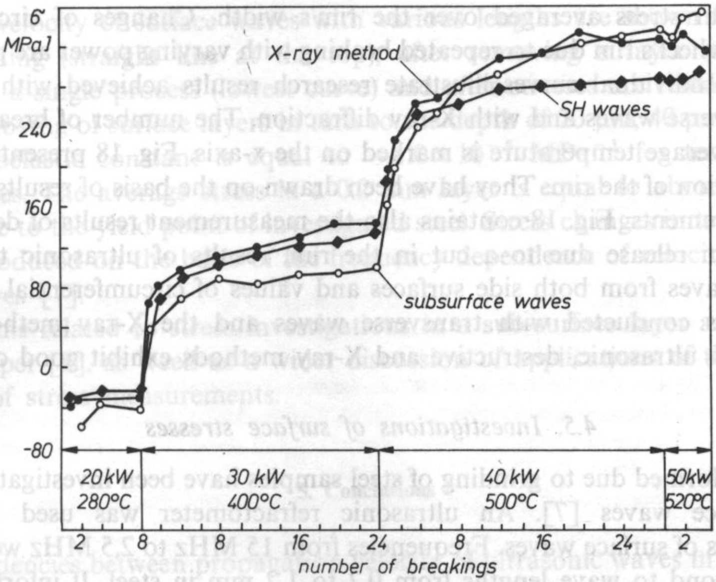

FIG. 17. Changes of circumferential stresses in the rim of a new wheel due to braking in a test banch. The number of brakings is marked on the horizontal axis and circumferential stress is marked on the vertical axis. The power of individual series of brakings and maximal temperature on the tread is noted under the curves

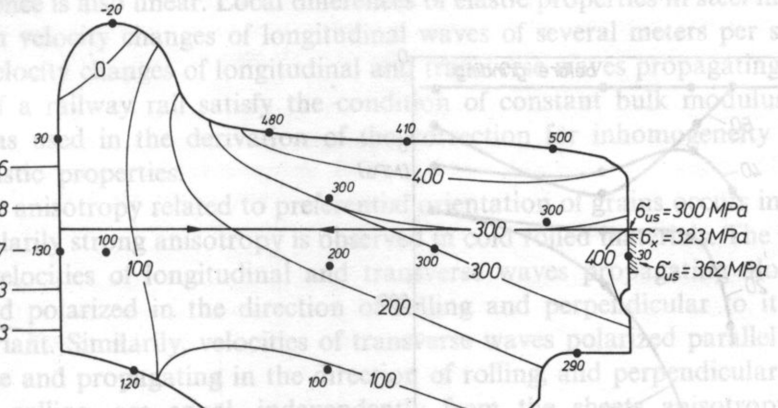

FIG. 18. Distribution of stress on the surface and in the cross-section of a wheel rim after a series of brakings. Comparision of ultrasonic, X-ray and destructive investigations

circumferential stress averaged over the rim's width. Changes of circumferential stress in the wheel's rim due to repeated braking with varying power are presented in Fig. 17 [6]. Individual curves illustrate research results achieved with subsurface waves, transverse waves and with X-ray diffraction. The number of breakings, their power and average temperature is marked on the x-axis. Fig. 18 presents isobars in the cross-section of the rim. They have been drawn on the basis of results of residual stress measurements. Fig. 18 contains also the measurement results of deformations resulting from release due to a cut in the rim, results of ultrasonic testing with subsurface waves from both side surfaces and values of circumferential stress from measurements conducted with transverse waves and the X-ray method. Results achieved with ultrasonic, destructive and X-ray methods exhibit good consistency.

4.5. Investigations of surface stresses

Stresses formed due to grinding of steel samples have been investigated with the aid of surface waves [7]. An ultrasonic refractometer was used in velocity measurements of surface waves. Frequencies from 15 MHz to 2,5 MHz were applied. They correspond to wave lengths from 0.2 to 1.2 mm in steel. If informations on properties of the material collected with the aid of surface waves concern a layer corresponding to one wave length, then in this case collected information concerning stresses was averaged from subsurface layers with thickness from 0.2 mm to 1.2 mm. Residual stresses were introduced into samples in the course of rough grinding without cooling. A layer with thickness 100 μm was removed. In Fig. 19 relative

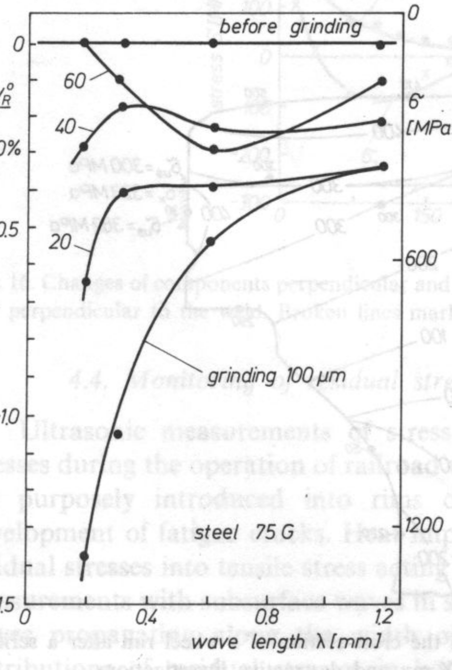

FIG. 19. Relative velocity changes of surface waves in terms of wave length λ for a steel sample before grinding (straight line on top of diagram), after rough grinding removing a 100 μm layer (lowest curve) and after a stressed layer with thickness 20, 40 and 60 μm, respectively, is removed

changes of velocity of surface waves with various lengths are shown for a sample before grinding (straight line at the top), after removing a layer with 100 μm thickness in a single process (lowest curve) and after stresses are removed through delicate removing of surface layers in turn to the depth of 20 μm, 40 μm and 60 μm. The acoustoelastic constant is equal to -1.0×10^{-5} MPa^{-1} for tested steel for surface waves. The average stress in a 0.2 mm layer is equal to about 1400 MPa, what is close to the yield point of investigated steel. Stress changes in terms of depth can be reproduced on the basis of the frequency dependence of velocity changes of surface waves [7].

Problems related to stress investigations in a sub-surface layer are studied in detail in paper [8], as weell as a wider discussion of applications of the ultrasonic technique of stress measurements.

5. Conclusions

Dependencies between propagation velocity of ultrasonic waves in construction materials and stress are linear. Velocity change due to stress is greatest in the case of longitudinal waves propagating in the direction of stress. A 10 MPa increment of tensile stress in steel causes a velocity decrease of longitudinal waves of about 0.75 m/s. Compression leads to a decrease of wave velocity. A temperature increase of steel of 1°C decreases the velocity of longitudinal waves by approximately 0.9 m/s. This dependence is also linear. Local differences of elastic properties in steel manifest themselves in velocity changes of longitudinal waves of several meters per second.

Local velocity changes of longitudinal and transverse waves propagating along the length of a railway rail satisfy the condition of constant bulk modulus. This condition was used in the derivation of the correction for inhomogeneity of the materials elastic properties.

Acoustic anisotropy related to preferential orientation of grains occurs in rolled steel. Particularily strong anisotropy is observed in cold rolled materials. The sum of squares of velocities of longitudinal and transverse waves propagating along the thickness and polarized in the direction of rolling and perpendicular to it is the texture invariant. Similarily, velocities of transverse waves polarized parallel to the sheets surface and propagating in the direction of rolling, and perpendicular to the direction of rolling, are equal, independently from the sheets anisotropy. The occurrence of stresses causes a departure from these laws and measurements of these deviations can be used in stress investigations. The size and distributions of texture in metallurgical products made with a reproducible production technology are the same. Subtracting from measured anisotropy, anisotropy related to texture we reach the value of anisotropy resulting from occuring stress, only. This dependence can be used in investigations of stress fields in rolled, drawn and cast elements.

Subsurface and bulk waves can be used in certain cases in the assessment of stress distributions in the volume of tested element. The application of surface waves with various frequencies makes it possible to investigate stress gradients in the direction perpendicular to the surface.

Comparative X-ray and destructive testing confirms the correctness of results of ultrasonic measurements of residual stresses.

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