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BRIEF NOTE

SPECIFIC PERFORMANCE OF IDT EDGE FINGERS

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A rigorous field theory has been developed that allows to analyze the generation and detection performance of each metal strip in a system of a few of them. The theory shows that the edge strips of the system works differently as compared with these positioned inside the system. The familiar δ-function model however, can still be applied provided that the δ-sources of SAW are properly scaled and shifted from their position at the centres of the strips. The scaling factor and the shift magnitude (which are larger for the edge electrodes of IDT) are determined by theory. Some numerical and experimental examples are presented, which seem to agree well.

1. Introduction

Surface acoustic wave (SAW) propagation in a system of metal strips deposited on a piezoelectric substrate occurs in a number of SAW devices:

in interdigital transducers (IDT) consisting of two or more metal strips (fingers), which generate and detect SAW,

in resonators, where metal strips can be used for SAW reflection,

in reflective array compressors (RAC), where the reflection of obliquely propagating SAW from metal strips is exploited,

in convolvers, where a strip is applied to form a waveguide for SAW.

Allowing certain idealization, i.e. neglecting mechanical and certain electrical properties of electrodes (mass, elasticity, finite electric conductance), the mathematical model of the above – mentioned system of metal strips corresponds to the boundary problem with mixed electrical, and homogeneous mechanical boundary conditions. In this work the method for analyzing SAW propagation in a system of metal strips proposed in [1] (for eigenvalue problems) and developed in [2, 3] (for nonhomogeneous problems concerning SAW generation and detection) is applied.

A generalization of the method [1] from the case of periodic split strips to the case of multiperiodic strips (the periodically repeated groups of several metal strips) is given as well as a functional dependence between currents and voltages of the electrodes. The transadmittance relation is derived, expressing the dependence of the current flowing to a given grounded metal strip on the potentials of the other strips. Numerical results for a group of several electrodes are presented, which show the dependence of electromechanical transformation efficiency of a metal strip on the strip position inside the group. These results allow one to take into account the specific performance of edge fingers of IDTs.

2. Formulation of the boundary problem

A piezoelectric half-space is considered. The surface acoustic wave propagation is described by a complex harmonic function

$$e^{j(\omega t - kx_3)} \tag{1}$$

where ω is an angular frequency (the factor $\exp(j\omega t)$ is neglected throughout the paper) and k is the wave-number of SAW (Fig. 1). Neglecting bulk waves, the approximation of the effective surface permittivity that describes sufficiently the piezoelectric halfspace is given by [1, 2]

$$\varepsilon(k) = -j\varDelta D_{\perp}/E_{\parallel} = \varepsilon_{ef}(k^2 - k_v^2)/(k^2 - k_0^2) \tag{2}$$

where k_0 and k_v are wave-numbers of SAW propagating on metalized and free halfspace, correspondingly. The strip width w and the separation $2p$ between strip centers are equal within every group of N strips, and the groups repeat periodically

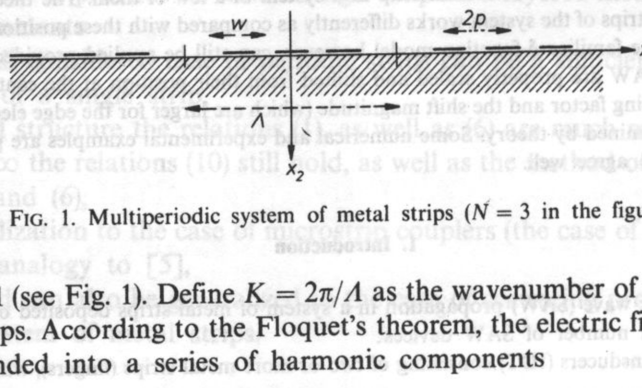

FIG. 1. Multiperiodic system of metal strips ($N = 3$ in the figure)

with period $\varLambda$ (see Fig. 1). Define $K = 2\pi/\varLambda$ as the wavenumber of the system of the groups of strips. According to the Floquet's theorem, the electric field in the system can be expanded into a series of harmonic components

$$E_{\parallel} = \sum_{n=-\infty}^{\infty} E_n \exp(-j(k+nK)x_3) \tag{3}$$

and similarly $\varDelta D_{\perp}$ (the difference of the electric flux density). The boundary problem can be formulated as follows:

$$E_{\parallel} = 0 \quad \text{on the strips,} \quad \varDelta D_{\perp} = 0 \text{ between the strips.} \tag{4}$$

Proceeding in a similar manner as in [1, 2] new functions have been derived

$$G_N(\theta; \alpha) = \sum_{n=-\infty}^{\infty} S_n X_n^N e^{-jn\theta} \quad F_N(\theta; \alpha) = \sum_{n=-\infty}^{\infty} X_n^N e^{-jn\theta} \tag{5}$$

which fulfill the corresponding conditions

$$G_N(\theta; \alpha) = 0 \text{ on the strips,} \quad F_N(\theta; \alpha) = 0 \text{ between strips,} \tag{6}$$

where parameter $\alpha = Kp$. Details concerning these functions see Appendix A.

Applying the method [1], the quantities $E_{\|}$ and $\Delta D_{\perp}$ can be expressed as

$$E_{\|} = \sum_{m=-M_1}^{M_2} \alpha_m G_N(Kx_3; Kp)e^{-jmKx_3}, \quad \Delta D_{\perp} = j\sum_{m=-M_1}^{M_2} \beta_m F_N(Kx_3; Kp)e^{-jmKx_3}. \tag{7}$$

3. Currents and potentials of electrodes

The relations (7) allows the calculation of the value of currents flowing to a single metal strip for any wavenumber k as follows

$$\mathbf{I}^k = -2\omega\varepsilon_{\text{ef}} \cdot \sum_{m=-M_1}^{M_2} \alpha_m I_m^k, \; I_m^k = \sum_{n=-\infty}^{\infty} \frac{X_{n-m}^N \sin[(k+nK)w/2]}{k+nK} e^{-j(k+nK)x_k} \tag{8}$$

where $x_k = (2k-N-1)p, (k = 1, 2, \ldots N)$ and similarly the l-th strip potential

$$\mathbf{V}^l = -j\sum_{m=-M_1}^{M_2} \alpha_m V_m^1, \quad V_m^l = \sum_{n=-\infty}^{\infty} \frac{S_{n-m} X_{n-m}^N}{k+nK} \cdot e^{-j(k+nK)x_1} \tag{9}$$

It is noticed, that α_m should satisfy some conditions to ensure that the potential is constant on every strip and the conditions that the strip potentials are $\mathbf{V}^l$ (see Appendix B for details). This allows us to express the unknowns α_m by the values of $\mathbf{V}^l$, which substituted to Eq. (8) gives

$$\mathbf{I}^k = \mathbf{Y}_{kl}\mathbf{V}^l, \; \mathbf{Y}_{kl} = -j2\omega\varepsilon_{\text{ef}} I_m^k [\mathscr{I}^{-1}]_m^l, \text{ (sum after } m, l). \tag{10}$$

4. Transadmittance relations

The relation (10) should be considered as the relation between Fourier transforms $\mathbf{I}^k$, $\mathbf{V}^l$ of discrete functions, namely the discrete set of strip currents I^k and the discrete set of voltages V^1, where indices k or l describe the position of the given strip on the x_3 axis. The transform parameter k is treated as independent variable taking its value from the area of first Brillouin zone $-K/2 < k < K/2$ [2]. The inverse Fourier transform of a discrete function is

$$F^l = \frac{1}{K} \int_{-K/2}^{K/2} \mathbf{F}^l(k) e^{-jkx_3} dk. \tag{11}$$

The following notations are introduced in order to determine the strips uniquely: $s = (m, k)$ and $t = (n, l)$ where k, l – number of metal strip in the group, $k, l \in [1, N]$, and n, m – number of the group, $n, m \in (-\infty, \infty)$. Applying (11) to (10) results in

$$I_s = Y_{st} V_t \quad \text{(sum after } t) \tag{12}$$

Table 1. Relative transformation efficiency b_{kl} of electrodes

N	l \ k	1	2	3	4	5	6	7
2	1	0.6674	0.4834 +j0.0130					
3	1	0.7825	0.7407 +j0.0035	0.6839 +j0.0102				
	2	0.7241 +j0.0122	0.8409	0.7241 +j0.0122				
4	1	0.8409	0.8168 +j0.0012	0.7708 +j0.0011	0.7390 +j0.0026			
	2	0.7772 +j0.0068	0.9036	0.8698 +j0.0013	0.7917 +j0.0053			
5	1	0.8408	0.8168 +j0.0012	0.7934 +j0.0004	0.7707 +j0.0015	0.7392 +j0.0001		
	2	0.7772 +j0.0068	0.9036	0.8698 +j0.0013	0.8373 +j0.0001	0.7922 +j0.0003		
	3	0.8332 +j0.0052	0.9039 +j0.0014	0.9554	0.9039 +j0.0014	0.8332 +j0.0052		
6	1	0.8409	0.8171 +j0.0008	0.8168 +j0.0016	0.7932 +j0.0031	0.7704 +j0.0041	0.7391 +j0.0024	
	2	0.7772 +j0.0068	0.9036	0.8824 +j0.0009	0.8698 +j0.0016	0.8372 +j0.0027	0.7924 +j0.0005	
	3	0.8332 +j0.0052	0.9039 +j0.0013	0.9999	0.9624 +j0.0001	0.9262 +j0.0017	0.8754 +j0.0004	
7	1	0.8409	0.8244 +j0.0002	0.8081 +j0.0022	0.7918 +j0.0041	0.7758 +j0.0059	0.7601 +j0.0075	0.7382 +j0.0076
	2	0.7772 +j0.0011	0.9036	0.8826 +j0.0003	0.8624 +j0.0021	0.8410 +j0.0041	0.8210 +j0.0057	0.7385 +j0.0076
	3	0.8320 +j0.0021	0.9036 +j0.0002	0.9554	0.9271 +j0.0001	0.9017 +j0.0021	0.8752 +j0.0037	0.7933 +j0.0056
	4	0.8752 +j0.0005	0.9263 +j0.0017	0.9612 +j0.0001	0.9999	0.9612 +j0.0001	0.9263 +j0.0017	0.8752 +j0.0005

where

$$y_{st} = -j\frac{2}{K}\cdot\omega\cdot\varepsilon_{ef}\int_{-K/2}^{K/2} R_{kl}e^{j(x_s-x_t)k}dk, \quad R_{kl} = \sum_{m=-M_1}^{M_2} I_m^k[\mathcal{J}^{-1}]_m^l. \tag{13}$$

The integral (13) can be evaluated partly numerically and partly using the Cauchy theorem. Finally, the result of integration can be written as

$$y_{st} = y_{st}^R + y_{st}^C, \quad y_{st}^R = b_{kl}[(\Delta v/v)s(1-s)\sin\pi s]e^{-j|x_s-x_t|r_0} \tag{14}$$

where $s = pr_0/\pi$ and r_0 is the wave number of SAW propagating in periodic metal strips with period $2p$. Index C describes terms connected with mutual capacitance of the metal strips. The most interesting term Y_{st}^R, which is the radiation part of transadmittance, is proportional to the parameter characterizing the piezoelectricity of the substrate $(\Delta v/v)$. We recognize the term describing time delay of SAW between the strips s and t in the form $\exp(-j|x_s-x_t|r_0)$, however, b_{kl} can have complex value so that the phase shift between I_s and V_t is described by both the above exponent term and $\arg\{b_{kl}\}$ simultaneously. The values of b_{kl} for a group of N electrodes is shown in Table I for the center frequency $f_0 = v/4p$. It is to be noted that the dependence of b_{kl} on frequency is week so that the values given in Table I can be used for a wide frequency band about f_0.

5. Theory of IDT

Below we consider strips having their widths equal to the spacings between them in the group, and the period of groups is assumed so large, that groups are almost isolated. This models the single group of N metal strips deposited on a piezoelectric substrate, that is exactly the case of interdigital transducer. The result (14) can be

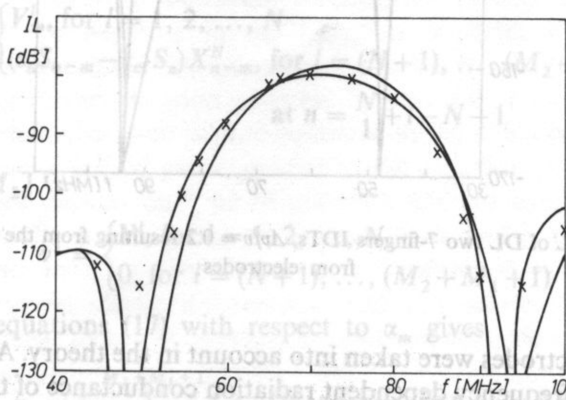

FIG. 2. IL of SAW DL on YX SiO_2 composed of two 7-fingers IDTs: — the above theory, — — — the δ-function model [2], + + + experimental data (thanks to Dr J. FILIPIAK and Dr A. KAWALEC for experimental data)

easily applied in analysis of interdigital transducers [3]. To do this, one should take into account that some transducer fingers are connected to one transducer bar, while the others are connected to the other bar. Correspondingly, the finger potentials are V_1 and V_2. The currents of transducer bars can thus be evaluated on the strength of Eqs. (12) and (14), and finally, the potentials V_1 and V_2 results from the Kirchoff's laws [4]. Fig. 2 shows the results of analysis of delay line consisting of IDTs having 7 fingers, aperture $A = 4$ mm and $p = 11$ μm. *YX* quartz was applied as the *DL* substrate. Shown are results given by the present theory, the theory [2] for periodic fingers (that is equivalent to δ-function model) and the experimental results, for comparison. The main effect of the specific performance of the each fingers of *DL* transducers (that is the different b_{kl} for different k and l) is the broadenning of *DL* passband and the increasing of *DL* insertion loss, as compared with [2] or δ function model. The results presented in this paper are quite similar to those in [5] however they were obtained on rigorous field theory of propagation and generation of SAW under periodic system of groups of electrodes. Such a second-order effect, as SAW

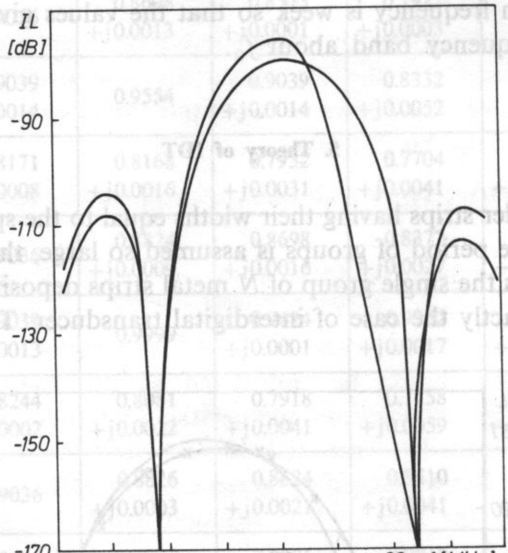

FIG. 3. Distortion of IL of DL two 7-fingers IDTs, $\Delta v/v = 0.2$ resulting from the $\Delta v/v$ reflection of SAW from electrodes

reflection from electrodes were taken into account in the theory. As known [3, 6] this effect distorts the frequency dependent radiation conductance of the transducer. This is illustrated on Fig. 3. The same delay line as presented above was analyzed and $\Delta v/v$ was applied as high as 0.2 in order to make the distortion clearly seen. To our best knowledge, this is the first rigorous theory of that effect.

Appendix A

An expansion of G_N or F_N into the Fourier's series gives

$$G_N(\theta;\alpha) = \sum_{n=-\infty}^{\infty} S_n X_n^N e^{-jn\theta}, \qquad F_N(\theta;\alpha) = \sum_{n=-\infty}^{\infty} X_n^N e^{-jn\theta} \tag{15}$$

where

$$\begin{cases} & \text{for } n \geqslant 0 & & \text{for } n < 0 \\ X_n^1 = P_n(\cos\Delta), & & X_{-n}^N = X_{n-1}^N \text{ for odd } N \\ X_n^2 = \sum_{m=0}^{n-1} P_m P_{n-m-1}\cos[(2m-n+1)\alpha], & X_0^2 = 0 & X_{-n}^N = -X_n^N \text{ for even } N \\ \dots\dots\dots & & \\ X_n^N = 2\sum_{m=1}^{n} X_m^2|_{(N-1)\alpha} \cdot X_{n-m}^{N-2}, & & \end{cases} \tag{16}$$

where $X_m^2|_\beta$ means that X_m^2 are the expansion coefficients of $F_2(\theta;\beta)$.

Appendix B

Assuming, that $\mathbf{V}^l$ are known, the following set of equations is obtained

$$\sum_{m=-M_1}^{M_2} \mathscr{I}_m^1 \alpha_m = jv^l, \qquad l = 1, 2, \dots, (M_2+M_1+1) \tag{17}$$

where

$$\mathscr{I}_m^1 = \begin{cases} V_m^l, \text{ for } l = 1, 2, \dots, N \\ (\varepsilon_n S_{n-m} - \varepsilon_{ef} S_n) X_{n-m}^N, \text{ for } l = (N+1), \dots, (M_2+M_1+1) \\ \qquad \text{at } n = \frac{N}{1} + l - N - 1 \end{cases} \tag{18}$$

for $m \in [-M_1, M_2]$

$$v^1 = \begin{cases} \mathbf{V}^l, \text{ for } l = 1, 2, \dots, N \\ 0, \text{ for } l = (N+1), \dots, (M_2+M_1+1) \end{cases} \tag{19}$$

Solution of the equations (17) with respect to α_m gives

$$\alpha_m = j \cdot \sum_{n=1}^{M_2+M_1+1} [\mathscr{I}^{-1}]_m^n v^n, \; m = -M_1, \dots, M_2 \tag{20}$$

where $\mathscr{I}^{-1}$ is the inverse matrix to $\mathscr{I}$, what gives next (10).

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