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# BRIEF NOTE

## NEW APPROACH IN THE THEORY OF MICROSTRIP WAVEGUIDES

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In literature, the Galerkin's method is usually applied in analysis of microstrip waveguides and couplers. Below, a new method is proposed that is a generalization of one developed previously for analysis of SAW waveguides [1] to the electromagnetic case. The proposed approach exploits some identities fulfilling by Fourier series which coefficients are expressed by Legendre functions.

### 1. Formulation of the problem

Let's consider a periodic system of metal strips on a plane $y = 0$ in vacuum. The period of strips is $2\pi/K$ and they have infinite length along $z$-axis. Following the Floquet theorem the solution to the Maxwell equations for the system considered has a form (A means an electromagnetic field component)

$$A_n^{\pm} e^{j\omega t} e^{-jr_n x} e^{-js_n^{\pm} y} e^{-jkz} \tag{1.1}$$

$$r_n = r + nK,\; k_0 = \omega^2 \mu_0 \varepsilon_0 \tag{1.2}$$

$$s_n^{+} = -s_n^{-} = (k_0^2 - k^2 - r_n^2)^{1/2} \tag{1.3}$$

where indices $+$ and $-$ mark the solution in the upper ($y > 0$) and lower ($y < 0$) halfspaces where they fulfill the radiation condition.

Introducing Hertz potentials $\hat{z}\Phi$ and $\hat{z}\Psi$, the electromagnetic field components can be expressed as follows [2]

$$E_x = -\omega\mu_0 s_n \Phi_n - kr_n \Psi_n \tag{2.1}$$

$$E_z = (k_0^2 - k^2)\Psi_n \tag{2.2}$$

$$H_x = -kr_n \Phi_n + \omega\varepsilon_0 s_n \Psi_n \tag{2.3}$$

$$H_z = (k_0^2 - k^2)\Phi_n \tag{2.4}$$

where index $n$ describes the wave-components correspondingly to (1).

The boundary conditions at the plane $y = 0$ are

$$E_x^+ = E_x^- = E_x \quad \text{and} \quad E_x = 0 \quad \text{on metal strips} \tag{3.1}$$

$$E_z^+ = E_z^- = E_z \quad \text{and} \quad E_z = 0 \quad \text{on metal strips} \tag{3.2}$$

$$H_x^+ - H_x^- = 0 \quad \text{between strips} \tag{3.3}$$

$$H_z^+ - H_z^- = 0 \quad \text{between strips} \tag{3.4}$$

**2. Construction of the solution**

There are known identities [3], [4]

$$\sum_{n=-\infty} e^{-jnKx} \sum_{m=M_1}^{M_2} \alpha_m P_{n-m}^{\mu} = 0, \quad \text{between strips} \tag{4.1}$$

$$\sum_{n=-\infty} e^{-jnKx} \sum_{m=M_1}^{M_2} \beta_m S_{n-m} P_{n-m}^{\mu} = 0, \quad \text{in area covered by strips} \tag{4.2}$$

where $P$ denotes Legendre functions of argument $\Delta = \cos(Kw/2)$, where $w$ is the strip width, and $\mu \leqslant 0$, $S_k = 1$ for $k \geqslant 0$ or $S_k = -1$ for $k < 0$. There are arbitrary coefficients $\alpha$ i $\beta$, which allow to express any function in a form of (4.1) or (4.2) in the proper domain under the strips or between the strips, if only this function behaves at the strip edges like $\varepsilon^{-1/2-\mu}$ for $\varepsilon \to 0$.

Then, taking into account (3.2) and (3.1) we can write

$$\Psi_n^+ = \Psi_n^- = \Psi = \Psi_m^{\mu} S_{n-m} P_{n-m}^{\mu} \tag{5.1}$$

$$\Phi_n^+ = -\Phi_n^- = \Phi_n = \phi_m^{\nu} P_{n-m}^{\nu} \tag{5.2}$$

where (5.2) ensure fulfilling (3.4).

The relations (3.1) and (3.3) require fulfilling

$$E_x = -\omega\mu_0 s_n \Phi_n - kr_n \Psi_n = e_m^{\mu} S_{n-m} P_{n-m}^{\mu} \tag{6.1}$$

$$\frac{1}{2}[H_x] = -kr_n \Phi_n - \omega\varepsilon_0 s_n \Psi_n = h_m^{\nu} P_{n-m}^{\nu} \tag{6.2}$$

for every $n$ (above, summation convention concerns $\mu$, $\nu$, $m$ only). The solution of (6) should have a form of (5), that is

$$\phi_m^{\nu} P_{n-m}^{\nu} = \frac{e_m^{\mu} \omega\varepsilon_0 s_n S_{n-m} P_{n-m}^{\mu} + h_m^{\nu} kr_n P_{n-m}^{\nu}}{-\omega^2 \varepsilon_0 \mu_0 s_n^2 - k^2 r_n^2} \tag{7.1}$$

$$\psi_m^{\mu} S_{n-m} P_{n-m}^{\mu} = \frac{e_m^{\mu} kr_n S_{n-m} P_{n-m}^{\mu} - h_m^{\nu} \omega\mu_0 s_n P_{n-m}^{\nu}}{\omega^2 \varepsilon_0 \mu_0 s_n^2 - k^2 r_n^2} \tag{7.2}$$

The idea of the method [1] is closed in requirement, that the above equations are identities for every $|n| \to \infty$, in approximation for every $n < N_1 < 0$ and for every $n > N_2 > 0$, where $s_n \approx -j|r_n|$ (this approximation makes (7) much simpler). In the remaining area, that is for $N_1 \leqslant n \leqslant N_2$ the relations (7) will be satisfied through properly chosen coefficients $\phi$, $\psi$, $e$ and $h$.

Consider the area $n < N_1$ or $n > N_2$.

Taking the approximation $s_n \approx -j|nK+r|$ into account and the identity [3]

$$\left(n+\frac{r}{K}\right)P_{n-m}^{(-1)}(\Delta) = \left(m-1/2+\frac{r}{K}\right)P_{n-m}^{(-1)}(\Delta)+\frac{P_{n-(m+1)}^{(0)}(\Delta)-P_{n-(m-1)}^{(0)}(\Delta)}{2(1-\Delta^2)^{1/2}} \tag{8}$$

as well as limiting the representation (5) to the necessary $\mu$ and $\nu$, that is in our problem to

$$\phi_m^{(-1)} = \phi_m, \psi_m^{(-1)} = \psi_m \tag{9}$$

only, the relations (7), taken as identities fulfilled for every n in the considered limits, result in

$$e_m^{(-1)} = \left(m-1/2+\frac{r}{K}\right)(j\omega\mu_0\phi_m-k\psi_m) \tag{10.1}$$

$$h_m^{(-1)} = \left(m-1/2+\frac{r}{K}\right)(-k\phi_m-j\omega\varepsilon_0\psi_m) \tag{10.2}$$

$$e_m^{(0)} = [j\omega\mu_0(\phi_{m-1}-\phi_{m+1})-k(\psi_{m-1}-\psi_{m+1})]/[2(1-\Delta^2)^{1/2}] \tag{10.3}$$

$$h_m^{(0)} = [-k(\phi_{m-1}-\phi_{m+1})-j\omega\varepsilon_0(\psi_{m-1}-\psi_{m+1})]/[2(1-\Delta^2)^{1/2}] \tag{10.4}$$

It means that we reduced the unknowns to $\phi$ i $\psi$ only.

Consider now the relations (7) for $n \in [N_1, N_2]$.

The unknows $\phi$ i $\psi$ (they are vectors of $M_2-M_1+1$ dimension) should be chosen from the condition (7) taken for every $n$ in the limits considered. This requires to apply the number of unknows equal the number of equations that is $M_2-M_1+1 = N_2-N_1+1$.

### 3. An example

The purpose of this example is not to solve any particular problem, but to show the way, the solution can be obtained. Let us then apply the simplest case of $K=1$, $k_0 = 1$, and $\Delta = 0$, in which $P_{-1}^0 = P_0^{-1} = P_0^0 = 1$, $P_1^0 = 0$, and assume that the wave propagates along $z$-axis, that is $r=0$ in (1). What is more, we apply the approximation that $s_n = |n|$ for every $n \neq 0$. Then in our disposal remains one $n$ (namely $n=0$), for which (7) should be separately considered as the condition for determining $\phi$ and $\psi$. In this circumstance it is sufficient to allow unknows $\phi_0$ and $\psi_0$ only in (6).

The relations (10) results in

$$e_c^{(-1)} = -(j\omega\mu_0\phi_0 - k\psi_0)/2 \qquad e_{-1}^{(0)} = (j\omega\mu_0\phi_0 + k\psi_0)/2 = -e_1^{(0)}$$

$$h_c^{(-1)} = (k\phi_0 + j\omega\varepsilon_0\psi_0)/2 \qquad h_{-1}^{(0)} = (k\phi_0 + j\omega\varepsilon_0\psi_0)/2 = -h_1^{(0)}$$

what substituted into (7) taken for $n = 0$ gives

$$\begin{bmatrix} j[(k^2-1)^{1/2}-1] & k\omega\varepsilon_0 \\ -k\omega\mu_0 & -j[(k^2-1)^{1/2}+1] \end{bmatrix} \begin{bmatrix} \phi_0 \\ \psi_0 \end{bmatrix} = 0$$

Finaly, the dispersion relation is $k = 1$. In this simplest case and under the approximation applied we obtained trivial solution.

## 4. Conclusions

It has been shown how the solution can be constructed for the case of periodic metal strips. But in practice we have a single strip on a layered media and this needs following comments

– for $K$ small considered case of periodic strips can be a sufficient approximation for the case of a single strip,

– for layered structure the relations (1), as well as (6) are much more complicated, but for $|n| \to \infty$ the relations (10) still hold, as well as the method of constructing the solution (5) and (6),

– the generalization to the case of microstrip couplers (the case of few strips) is also possible, by analogy to [5],

– the method can also be generalized to the case of scattering problem of wave on a periodic system of metal strips.

## References


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