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BRIEF NOTE

SAW GENERATED ELECTRIC CHARGE ON A METAL DISK ON PIEZOELECTRICS

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Scattering of a plane SAW by a circular, perfectly conducting disk on a piezoelectric halfspace is considered. A new theoretical approach allows to take into account the electric anisotropy of the substrate and gives the solution for the electric charge density on the disk in a form of fast convergent series for a wide range of wavelength of the SAW.

1. Introduction

Scattering of surface acoustic wave (SAW) by metal disk is exploited in some SAW devices [1], there is also an interest in considering the problem, as it may be helpful in modelling narrow interdigital transducers (IDT) [2].

There are many papers in literature considering electromagnetic wave scattering by a metal disk [3, 4], but they cannot be directly applied to the case of SAW for two reasons:

– in the SAW case the total current flowing to a grounded metal disk is of our major interest for wide range of wavelengths as compared with disk diameter.
– usually, the piezoelectric halfspace must be considered as an highly anisotropic dielectric.

The proposed theoretical approach is based on the known Galerkin's method, but further considerations are quite different and allow evaluation of
– the total current flowing to the grounded metal disk,
– the disk capacitance (anisotropy of the substrate accounted),
– the distribution of electric charge density on the disk in a form of fast convergent series that allows evaluation of the scattered acoustic field outside the disk.

2. The diffraction problem for a circular disk

We consider electric interaction only between the metal disk and SAW with angular frequency ω and wave-number k. The Green's function for a stress-free piezoelectric halfspace expressing the electric potential as dependent on electric charge, both on the substrate surface, can be divided into two terms, the first one describing dielectric properties of the substrate and the second one describing elastic waves, weakly coupled with electric quantities [5, 6].

Let $\mathbf{r}$ be the distance between the point (r, ϑ) where the potential is evaluated and the point (r', ϑ'), where electric charge is placed on the substrate surface (we apply the cylindrical coordinates r, ϑ, z but we confine our considerations to the substrate surface $z = 0$. The ϑ and ϑ' are angular coordinates describing the direction of the corresponding vector on the anisotropic halfspace). It can be shown, that the first above-mentioned dielectric term of the Green's function [7]

$$G(\mathbf{r}, \boldsymbol{\vartheta}) = [2\pi \mathbf{r} \varepsilon_0 \varepsilon_e (\boldsymbol{\vartheta} + \pi/2)]^{-1} \tag{1}$$

is much larger than the second term, if only $k\mathbf{r}K^2 \ll 1$, where K^2, which is usually less than 0.01, describes the piezoelectricity of the substrate. Thus, confining our consideration to disks having diameters of up to about 10 wavelengths, the second term of the Green's function can be neglected. The term $\varepsilon_0\varepsilon_e$, which is the effective surface permittivity dependent on ϑ for anisotropic piezoelectrics, can be written as follows (n-even)

$$(\varepsilon_0 \varepsilon_e)^{-1} = \sum \chi_n e^{jn\vartheta} \tag{2}$$

Under this simplification the condition that the grounded disk potential is equal to zero takes a form (the disk radius is applied equal to 1, for simplification, S – the disk area)

$$-\Phi^0(r, \vartheta) = \iint_S G(\mathbf{r}, \boldsymbol{\vartheta}) \varrho(r', \vartheta') dS \tag{3}$$

where ϱ is an electric charge distribution on the disk, and

$$\Phi^0 = \exp(jkr \sin \vartheta)$$

is the electric potential coupled with the impingent SAW.

3. Evaluation of charge distribution

Let apply the representation for the charge distribution on the disk in the form (m and n have the same parity, $n \geqslant 0$)

$$\varrho = \sigma_{mn} r^n (1 - r^2)^{-1/2} e^{jm\vartheta} \tag{4'}$$

After some transformations, the right-hand side of (3) can be written in the form (p, s, l – integers, J – Bessel function)

$$\sigma_{pn}\chi_{m-p}[(-1)^l A_s^{m,n}/l^s+(+1)^l B_s^{m,n}/l^s]J_m(\pi rl)e^{jm\vartheta} \tag{5}$$

where $0 \leqslant s \leqslant n$, and there is summation over l in (5), $0 < l < \infty$.

The key observation is, that the second component in the bracket above leads to the inappropriate solution for electric potential under the disk. In fact, it leads to the function like a cone [8]. To avoid such a solution we have to confine the representation (4′) of charge distribution as follows (Appendix)

$$n \geqslant |m| \tag{4''}$$

which is the sufficient condition for vanishing of B in (5).

The left-hand side of (3) can be written in the form

$$\frac{-\sin k}{k}\sum_l (-1)l\frac{(k/\pi l)^t}{1-(k/\pi l)^2}e^{jm\vartheta}J_m(\pi rl); \quad \begin{matrix} t=2 & \text{for} & m=0 \\ t=|m| & \text{for} & m\neq 0 \end{matrix} \tag{6}$$

In spite of the similarity of both expressions (5) and (6) we can not compare them directly for each l. To do this we have to use Lagrange extrapolation formula, allowing (6) to be approximated as follows $(0 \leqslant s \leqslant N)$

$$(-1)^l C_s^m/l^s e^{jm\vartheta}J_m(\pi rl) \tag{7}$$

if $k < N\pi$, where N is the upper limit of n, applied in the representation (4).

Now, we can compare (5) and (7) for each s-components separately, which results in the set of simultaneous equations for σ_{mn}. There is a triangular set of equations for each m, each set coupled with the other by the term χ_{m-p}. However note, that for isotropic substrate only χ_0 is different from zero, which allows solution of the equations for each m separately.

4. Evaluation of total electric charge generated by SAW

Integration of the charge distribution given in (4) results in the expression for a total charge on the disk of radius R

$$Q = 4(\sin kR/k)\chi_0^{-1} \tag{8}$$

and it follows from the above relation (apply $k=0$) that the disk capacitance is equal to (see [9] for isotropic case, where $\varepsilon_e = 1+\varepsilon$)

$$C = 4R/\chi_0 \tag{9}$$

Both relations are correct for arbitrary anisotropic halfspace, where χ_0 is described by the relation (2).

The above formula can be generalized to the case of an elliptic metal disk by

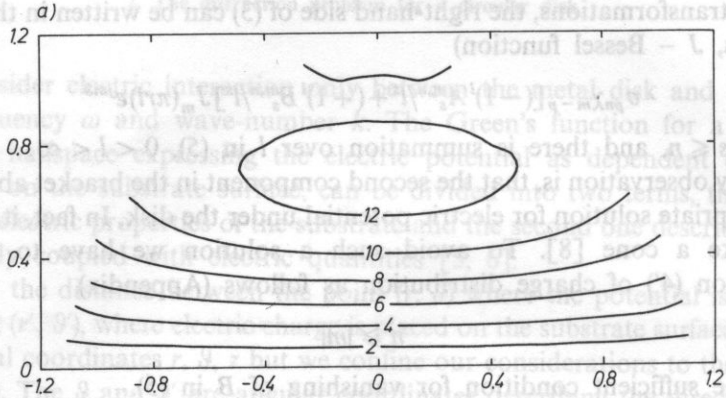

FIG. 1. The diagram of charge distribution for isotropic (a) and anisotropic (b) case. Only the imaginary component of a complex valued charge amplitude is shown, with square root term dropt in (4). The contour line levels are in arbitrary units

introducing the coordinate transformation of an ellipse to the circle. The transformation results in the dependence of ε_e on ϑ, which may become a quite complicated function, so that χ_0 changes, as well.

As a numerical example, Fig. 1 shows the diagram of charge distribution (imaginary part of its complex value), with square root term dropped in (4), for a circular disk on isotropic substrate, and for 4:1 elliptic disk on YZ lithium niobate, the major axis of the disk is rotated about 47 degrees off the Z axis (this orientation is applied for reflecting SAW from Z to X direction). In the latter case the figure shows the charge distribution in the coordinate system transformed as discussed above, so that the charge distribution on the equivalent circular disk is presented. We see considerably different charge distributions in these cases.

Appendix

Consider a charge distribution in somewhat different form

$$\varrho(r, \vartheta) = \sigma_{mn}\frac{T_n(r)}{(1-r^2)^{1/2}}e^{jm\vartheta} \tag{10}$$

which substituted to the right side of (3) yields the following expression for it (after many transformations)

$$\begin{gathered}\frac{1}{2}\sigma_{kn}\,\chi_{m-k}\,c_l^{(k,n)}(\operatorname{sign}k)^k e^{jm\vartheta}J_{|m|}(\pi l r)\\ c_l^{(m,n)} = (\pi/2)^2(\operatorname{sign}m)^m\left[\frac{J_{|m|-n+1}}{2}\frac{J_{|m|+n-1}}{2}+\frac{J_{|m|-n-1}}{2}\frac{J_{|m|+n+1}}{2}\right]\end{gathered} \tag{11}$$

where the argument of the Bessel functions J is the $(\pi l/2)$, sign $(m) = +1$ for $m \geqslant 0$ or -1 for $m < 0$. Some further transformations concerning the Bessel functions above, which have half-integer indices, allows to obtain (5), as well as to prove the thesis in the line next to (4').

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