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ELECTROACOUSTIC ANALOGIES APPLIED TO ACOUSTIC OSCILLATOR ANALYSIS

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Analogies were, so far, rarely applied to acoustic oscillator analysis, however, their use turns out to be advantageous. It allows, due to comparison of analogous electric and acoustic oscillators operation, for better understanding of the process of oscillation maintenance, as well as, the interpretation of the circuits behaviour. Particular conclusions are presented relative to examples of a bowed string, and of a labial pipe oscillators.

1. Introduction

It is almost incredible that one of the most ancient acoustic devices invented by our prehistoric ancestors to produce musical sound, the flue pipe, furnishes, so far, an unsolved problem for scientists attempting to explain fully the involved mechanism of oscillation maintenance. A more recent example of a similar problem is delivered by the mechanism of bowed string oscillations. Meanwhile, there is a large family of electric oscillators, thoroughly investigated, with very well known characteristics. Why the experience gained in the domain of electric oscillators could not be applied to mechanic and acoustic self-oscillating devices?

The most probable negative answer bases on formal constraints of electroacoustical analogies, which are limited to linear elements only, while self-oscillating circuits contain, as a rule, nonlinear elements. Nevertheless, as shown beneath, the analogies can be enlarged, at least qualitatively, to the comparative analyses of oscillator circuits of the mechanical, acoustical and electrical nature.

First of all, a brief review of electroacoustical analogies is necessary, including some comments and enlargements of their usual applications.

2. Electroacoustic analogies

Analogies between mechanic, acoustic and electric quantities were studied already in XIX century. Many authors paid particular attention to that matter and numerous textbooks contain presentations of analogies, laid down as appropriate tables of corresponding quantities and equivalent circuits [5], [7], [12], [13]. Although the theory of analogies originated by Lord Kelvin and by Firestone remains a valid and exhaustive basis of those publications a care should be kept in their use, because of some intricacy in particular presentations, and even few mistakes cointained therein.

A most complete and inspiring presentation of that matter has been given by MALECKI [8], where he even enlarged the well known concepts on the domain of field quantities. Quoting here all his consideration would consume too much place, thus only an abridged information on analogies, indispensable for this article, is given beneath.

Formal analogies, i.e. those based on similarity of equations describing electrical, mechanical and acoustical phenomena, are usually denoted as a table of corresponding quantities. For the case of the motional (corrected, or Firestone's) analogies, in contrast to the dynamical (classical, or Kelvin's) ones, the following quantities can be listed, as quoted in the Table I.

Table I.
Motional analogies

Electrical		Mechanical		Acoustical Quantities	
voltage	[V]	velocity	[m/s]	volume-velocity	[m^3/s]
current	[A]	force	[N]	pressure	[N/m^2]
charge	[C]	force impulse	[Ns]	pressure impulse	[Ns/m^2]
flux	[Wb]	displacement	[m]	volume displacement	[m^3]
inductance	[H]	compliance	[s^2/kg]	compliance	[m^4s^2/kg]
capacitance	[F]	mass	[kg]	inertance	[kg/m^4]
resistance	[Ω]	conductance	[s/kg]	conductance	[m^4s/kg]
conductance	[S]	resistance	[kg/s]	resistance	[kg/m^4s]

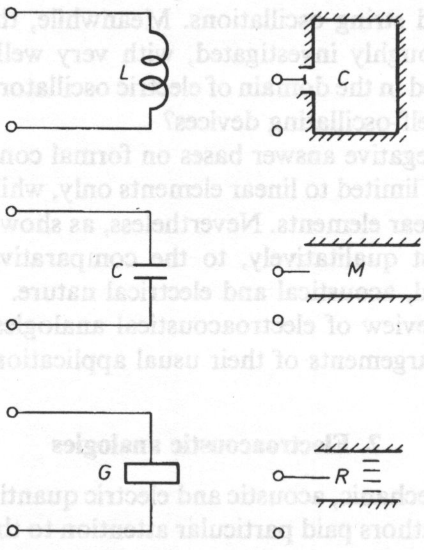

Fig. 1. Examples of motional analogy between two-poles.

Only the motional analogies, in contrast to dynamical ones, are considered here, because they keep unaffected topological features of the corresponding quantities. Moreover, they introduce impedances instead of admittances, and inversely. These properties are both favourable for a more clear presentation of the concept of analysis. Besides, keeping to one family of analogies reduces a probability of possible mistakes, numerous in practice, as mentioned above, even in textbooks.

Employing the correspondent quantities of the table given above, various equivalent circuits of the acoustical two-poles may be described as analogues to the electrical two-poles, e.g. those shown in Fig. 1, under assumption of linearity of all circuit elements.

3. Two-pole oscillator circuits

Further analogies may be considered, concerning oscillator circuits, composed of two-poles: the first one having non-linear maintenance characteristics, and the second one built of linear elements with resonant properties. Here, however, due to non-linearity of the maintaining two-poles the full analogy is possible only then, when shapes and scales of their non-linear characteristics are exactly analogous. Figure 2 shows the two basic types of equivalent circuits of electrical two-pole oscillators: the parallel,

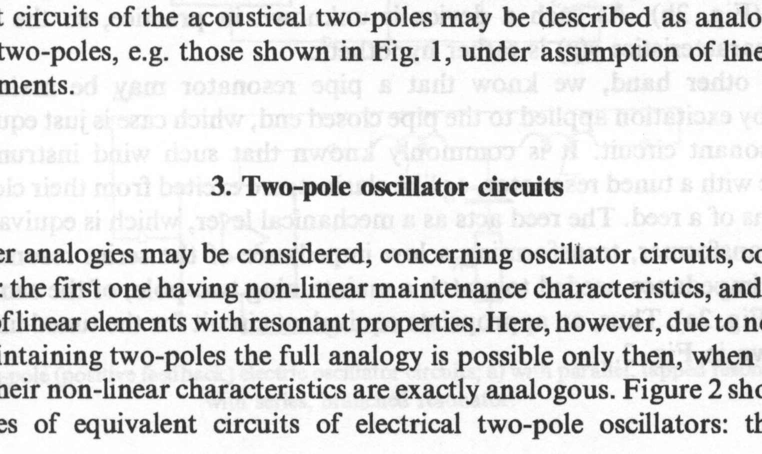

Fig. 2. Two basic types of two-pole oscillators
a) parallel controlled, b) controlled in series.

voltage-controlled oscillator, and the series, current controlled oscillator circuit, as well as their acoustical analogues. Mechanical analogies are omitted here, because their equivalent circuit are identical to acoustical ones, except differences in diagram symbols.

Even when strict analogy between non-linear two-poles is not reached, it may be sufficient to take advantage of the similarity of maintenance and of stability conditions of the oscillator circuit, as well as of its circuit variables.

A particular attention is to be paid to the two acoustical circuits shown in Fig. 2. While the parallel controlled oscillator represents the case of a labial pipe maintained by a jet action (Fig. 2a), which is a main topic of our consideration, the acoustical circuit controlled in series should require a special maintaining two-pole, pressure controlled (Fig. 2b). As such a device is unknown in practice, so the respective two-pole characteristics $v(p)$ is rather hypothetic.

On the other hand, we know that a pipe resonator may be maintained in oscillation by excitation applied to the pipe closed end, which case is just equivalent to a series resonant circuit. It is commonly known that such wind instruments like lingual pipe with a tuned resonator, or like clarinet, are excited from their closed ends, yet by means of a reed. The reed acts as a mechanical lever, which is equivalent to an electrical transformer, transforming a low impedance of the series resonant circuit into a high impedance needed to match a maintaining two-pole, of the same kind as that of the Fig. 2a). Thus, an appropriate equivalent circuit for the tuned lingual pipe is that shown in Fig. 3.

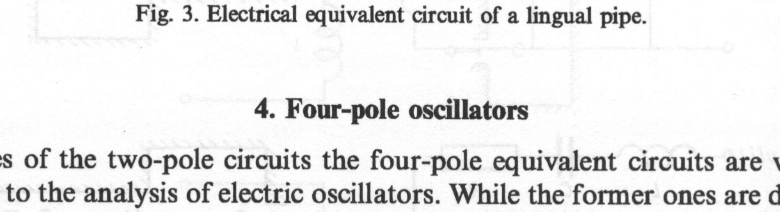

Fig. 3. Electrical equivalent circuit of a lingual pipe.

4. Four-pole oscillators

Besides of the two-pole circuits the four-pole equivalent circuits are very often employed to the analysis of electric oscillators. While the former ones are denoted as the negative imittance (i.e. either negative impedance or negative admittance) oscillators, the latter ones are called the positive feedback oscillators. The fourth, or at least the third, terminal of the equivalent circuit put out an appropriately phase-shifted variable from a divided or tapped resonant contour into the maintaining four-pole (at least three-pole), see Fig. 4.

Although many mechanical oscillators can be represented by four-pole equivalents, no acoustical examples of such circuits are known in practice, because acoustical resonators generally have neither taps nor branches. Therefore, attempts to analyze acoustical oscillators as feedback circuits, as e.g. is practized in edgetone theory, do not seem to be justified.

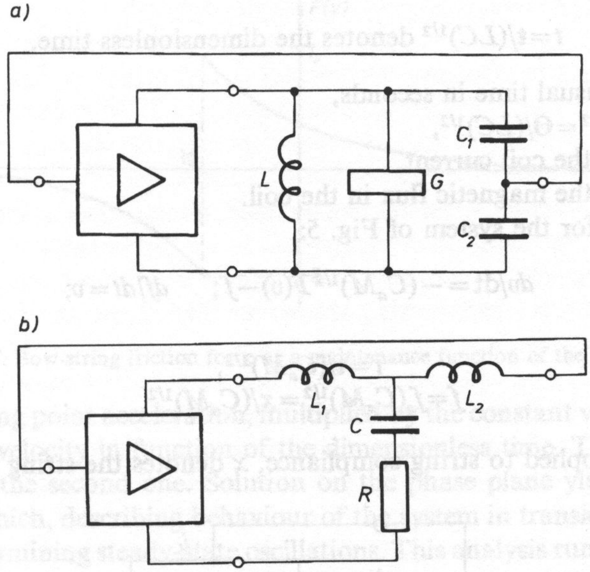

Fig. 4. Four-pole (positive feedback) electric oscillator circuits; a) with parallel, tapped resonator, b) with series, branched resonator.

5. Examples of oscillator analysis

5.1. Bowed string as a two-pole oscillator

As it was shown earlier [2], a bow-string system may be dealt with as a mechanical self-oscillating system with a resonant contour having one degree of freedom. The system, depicted in Fig. 5, may be analyzed as an equivalent electric analogue circuit shown in Fig. 6.

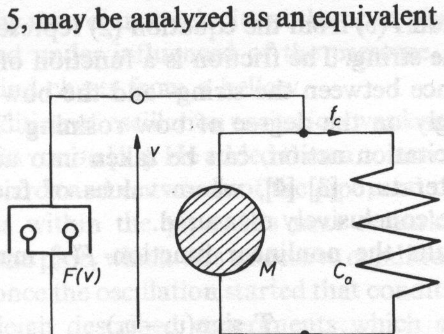

Fig. 5. Mechanical system representing a bow-string oscillator.

For the circuit of Fig. 6:

$$du/dt = -(L/C)^{1/2}\, I(u) - i\,; \qquad di/dt = u\,; \qquad (1)$$

where

$t = \mathrm{t}/(LC)^{1/2}$ denotes the dimensionless time,

whilst

t denotes usual time in seconds,

$i = i_L(LC)^{1/2} = \Theta/(LC)^{1/2}$,

i_L denotes the coil current,

Θ denotes the magnetic flux in the coil.

Analogically, for the system of Fig. 5:

$$dv/\mathrm{dt} = -(C_aM)^{1/2} F(v) - f; \qquad df/dt = v; \tag{2}$$

where

$$t = \mathrm{t}/(C_aM)^{1/2},$$
$$f = f_c(C_aM)^{1/2} = x/(C_aM)^{1/2},$$

f_c denotes force applied to string compliance, x denotes the string displacement.

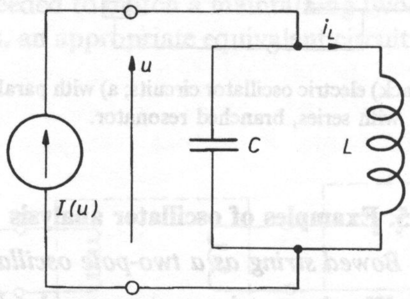

Fig. 6. Electrical circuit equivalent to the system of Fig. 5.

The nonlinear function $F(v)$ from the equation (2) represents the force of friction between the bow and the string. The friction is a function of the relative bow-string velocity, i.e. the difference between the string- and the bow-velocities $(v_s - v_b)$. The function depends strongly on the degree of bow rosining. This dependence, being essential for bowing excitation action, can be taken into account based on investigations reported in literature [3] [4], where values of friction force vs. relative bow-string velocity were conclusively measured.

Based on those results the nonlinear function $F(v)$ may be described by the following expression:

$$F(v) = \frac{T_o \operatorname{sign}(v_s - v_b)}{1 + k \mid v_s - v_b \mid}, \tag{3}$$

where

T_o denotes static friction force at $(v_s - v_b) = 0$

k is a coefficient depending on rosining.

The shape of function $F(v)$ is depicted in Fig. 7.

Substituting expression (3) into (2) gives a set of equations describing oscillations of the string point under the bow, on the phase plane. The variables of the set

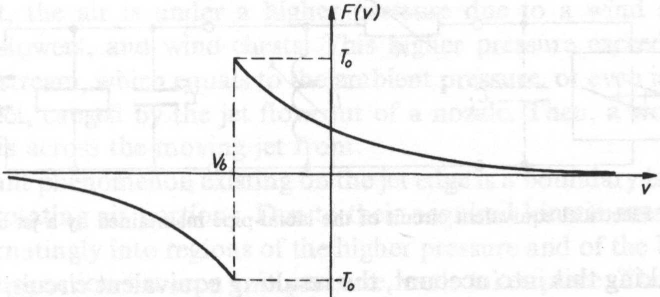

Fig. 7. Bow-string friction force as a maintenance function of the system.

represent the string point acceleration, multiplied by the constant value $(C_a M)^{1/2}$, and the string point velocity in function of the dimensionless time. The first variable is a differential of the second one. Solution on the phase plane yields trajectories of a phase point, which, describing behaviour of the system in transient states, tends to a limit-cycle determining steady-state oscillations. This analysis runs quite similarly to that one of an electric analogous oscillator.

5.2. *Labial pipe as a two-pole oscillator*

A sounding labial pipe may be represented by the acoustical equivalent circuit shown in Fig. 2 a). Theoretically the pipe resonator should be treated as a system with distributed constants, however, under usually accepted approximation it can be represented by a simple parallel equivalent circuit with lumped constants. When connected to a two-pole, which represents an air-jet maintenance action, the pipe resonator is controlled by the volume-velocity at the open pipe end, i.e. the air column at this end is accelerated under influenced of the pressure, delivered by the air jet, pumped through the wind chests from a bellow.

This topological condition of oscillation excitation was known from long ago. It was already stated by Lord Rayleigh [11]. He added then a comment concerning alternative deflections of the jet inwards and outwards of the pipe, and thought this motion to be maintaining oscillations within the pipe. His next remark concerned an accurate adjustment of the jet to the pipe, which was a decisive condition of oscillation onset. He noticed, however, that once the oscillation started that condition became less exacting.

Besides, Lord Rayleigh described experiments which showed that the natural frequency of the flue pipe resonator excited from an external source is lower than the frequency maintained by an usual blast.

Those remarks and, first of all, many experimental observations convince us, that a coupling two-pole is an unavoidable element of the equivalent circuit, which should represent mainly a compliance L', due to an influence of the static pressure exerted by the blast within the labium chamber, and a resistance R, due to losses of an air flow through the flue. Values of those coupling elements depend on operating conditions

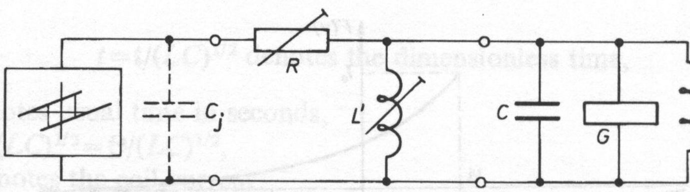

Fig. 8. Electrical equivalent circuit of the labial-pipe maintained by a jet action.

of the blast. Taking this into account, the resulting equivalent circuit, turned into its electrical form, adopts the following shape, see Fig. 8, where, moreover, an extra inertance C_j, marked with broken lines, has been added.

The inertance C_j is an analogue of the, so called, dynatron capacitance, a fictitious element playing an important role in the theory of oscillators. Under its influence, the operating frequency of a negative conductance, voltage controlled oscillator with simple resonant LGC circuit, is always lower than the natural frequency of the resonant circuit [6].

We have measured the operating frequency and the natural frequency for eight labial pipes of various types and pitches, and we have found all operating frequencies lower than natural ones. Thus, this property of acoustical oscillators is again analogous to electrical circuits.

Thanks to above proposed equivalent circuit configuration, the mentioned discrepant observations [11] or other similar measurement results [10] may be interpreted as occuring in circuits having low value of C_j and relatively high value of L'. In such circiuts the operating frequency may be indeed higher than the natural one. Moreover, the dependence of the compliance L' on wind pressure causes a reduction of the resulting circuit compliance with pressure increase, which, in turn, augments the operating frequency. Such mechanism explains why in acoustical oscillator the operating frequency increases with increasing wind pressure.

The jet action is nonlinear when its velocity is above certain limit value of the Reynolds number, which determines the turbulent air flow. The Reynolds number for a jet velocity v inside a pipe with a diameter d is:

$$\mathrm{Re} = \rho/\mu$$

where ρ/μ is the kinematic viscosity of the air.

Limit values to be outvalued as a turbulence condition, quoted in the literature, are inconsistent and discrepant. The resulting jet velocities, however, should be in the range not less than about several m/s.

The nonlinearity is an elementary condition for possible oscillation maintenance by a two-pole. It is difficult to describe its further properties, because of the dynamical character of the flow. However, assuming that an observer moves together with the jet, it is possible to create a simple, quasi-static velocity-pressure characteristics of the jet action.

Inside a jet, the air is under a higher pressure due to a wind supply system, composed of blowers, and wind-chests. This higher pressure exceeds the pressure around the jet stream, which equals to the ambient pressure, or even is lower, thanks to sucking effect, caused by the jet flow out of a nozzle. Then, a working pressure difference exists across the moving jet front.

An important phenomenon existing on the jet edge is a boundary layer, composed of vortices of rotating air portions. Due to their acquired kinetic energy of rotation, they enter alternatingly into regions of the higher pressure and of the lower one, then a rapid switching action between two pressure levels takes place. Those phenomena are, of course, transported in space forward with jet velocity, however, they may be treated as independent of their position in space, within a certain time, sufficiently long in comparison to several oscillation periods.

The main simplification of the jet action characteristics depends on an assumption of the plane front edge of the jet. Then only one coordinate is variable, while those representing the second and the third dimension, adopt constant values, thus, although ineffective in circuit description, they permit to keep the proper unit denominations for the corresponding quantities. None the less, the value of an air resistance, keeping its dimension, remains a real quantity.

Basing on those assumptions, a quasi-static characteristics of the jet action may be sketched, see Fig. 9. The two straight sections, marked with a heavy line, and denoted by H (for higher) and L (for lower pressure) represent the relation between pressure and particle velocity, for the two operational levels of static pressure.

Due to mentioned process of turbulence in the jet boundary layer, some portions of air at higher pressure are injected into the lower pressure region, from where, after a time, some portions are again injected into the higher pressure region. This was schematically depicted in Fig. 9. Such injection-switching may occur at velocities either slightly above the mean jest velocity, taken here as zero value on the abscissae axis, or equal, or slightly below it, so as it is shown on the diagram with broken lines.

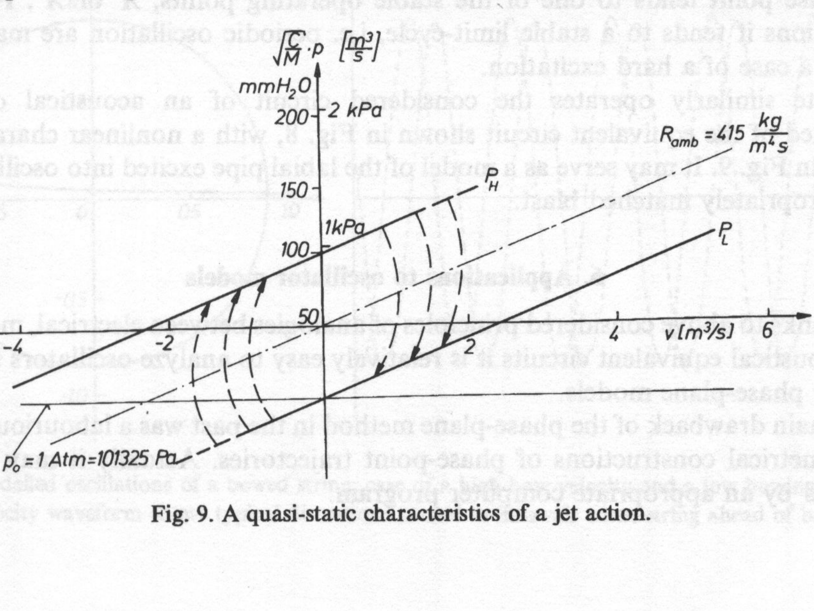

Fig. 9. A quasi-static characteristics of a jet action.

The derived, quasi-static characteristics of the jet action is quite similar to a static current-voltage characteristics of a two-pole, compound of a N-shaped, symmetric, negative conductance two-pole, with a resistance in series. The components and the resultant nonlinear characteristics are shown in Fig. 10. Authors investigated oscillators of this kind several years ago, and found them advantageous in various applications, due, mainly, to their stable operation [1].

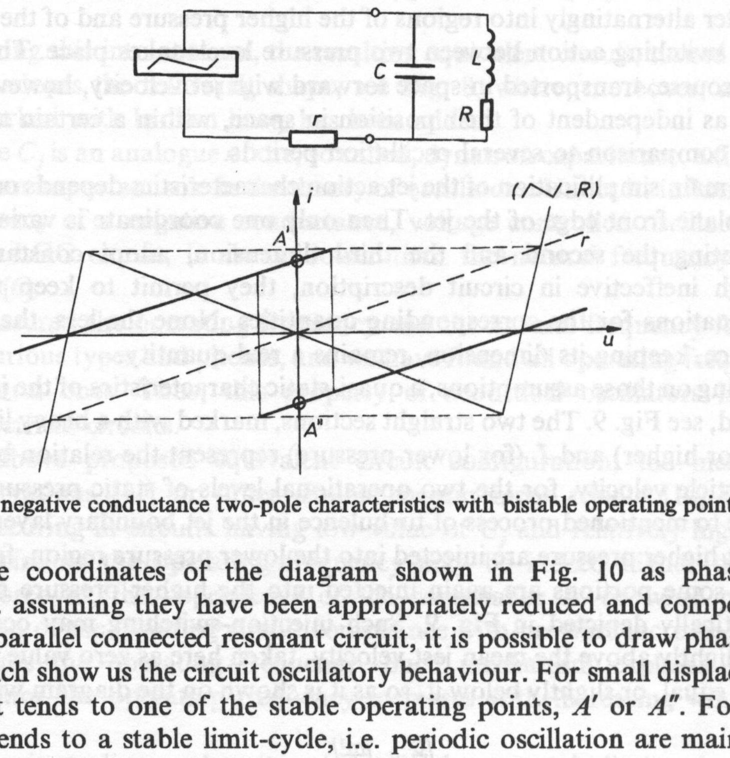

Fig. 10. A negative conductance two-pole characteristics with bistable operating point.

Treating the coordinates of the diagram shown in Fig. 10 as phase-plane coordinates, i.e. assuming they have been appropriately reduced and compensated, according to a parallel connected resonant circuit, it is possible to draw phase point trajectories, which show us the circuit oscillatory behaviour. For small displacements the phase point tends to one of the stable operating points, *A'* or *A"*. For larger elongations it tends to a stable limit-cycle, i.e. periodic oscillation are maintained. This is a case of a hard excitation.

Quite similarly operates the considered circuit of an acoustical oscillator, combined of the equivalent circuit shown in Fig. 8, with a nonlinear characteristics shown in Fig. 9. It may serve as a model of the labial pipe excited into oscillations by an appropriately matched blast.

6. Applications to oscillator models

Thanks to above considered principles of analogies between electrical, mechanical and acoustical equivalent circuits it is relatively easy to analyze oscillators by means of their phase-plane models.

A main drawback of the phase-plane method in the past was a labourious process of geometrical constructions of phase-point trajectories. Actually it may be easily replaced by an appropriate computer program.

Beneath, a few selected examples of the phase-plane solutions are presented. Models were executed by means of a PC AT computer with a special program written in the Turbo Pascal language. For calculations the Runge-Kutta method was applied.

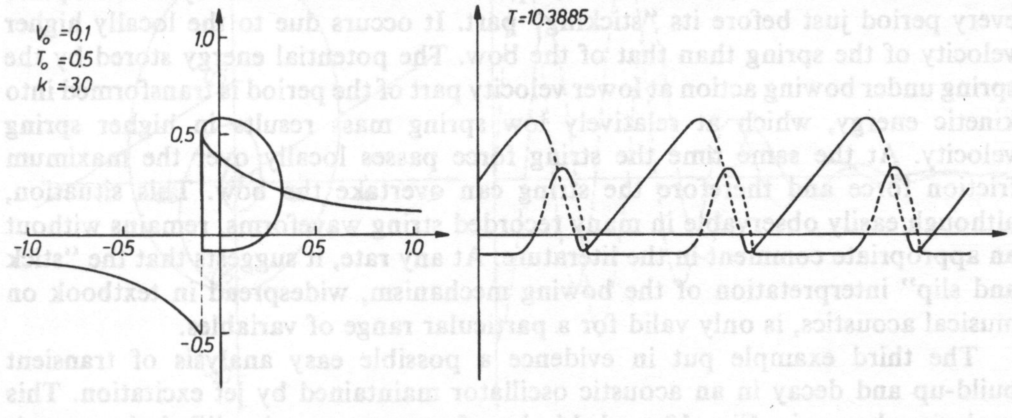

Fig. 11. Modelled oscillations of a bowed string; case of a bow velocity and a high bowing force;
— saw-tooth waveforms represent force vs. time,
— cut-off sines represent velocity vs. time.

The first example represents the case of string vibrations maintained by bow action at very low bowing velocity, see Fig. 11. Both waveforms, of velocity and of displacements are quite similar to those observed by many authors in such conditions, i.e. for a bow position very near to the bridge. A high value of applied bow force enhances the typical saw-tooth shape of displacement waveform.

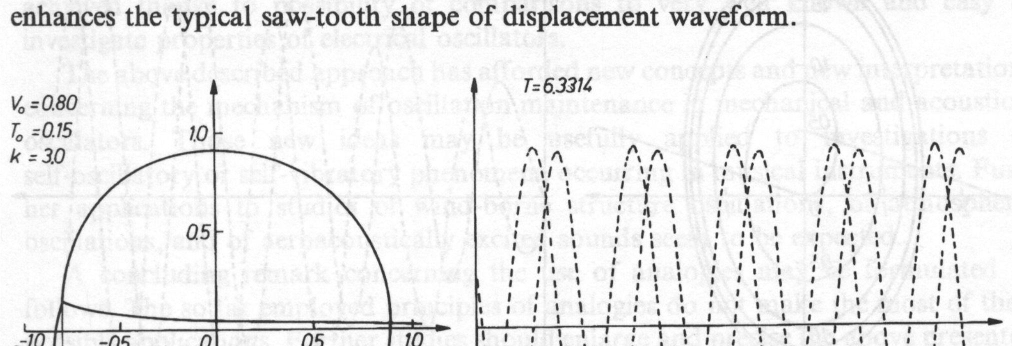

Fig. 12. Modelled oscillations of a bowed string; case of a high bow velocity and a low bowing force;
— velocity waveform shows typical distortions caused in intervals when string ahead of bow.

The second example concerns the conditions extremely different from the first one, see Fig. 12. The bow force is smaller at high bowing velocity. The waveforms result of almost sinusoidal shape. The "sticking" parts of period became very short, almost unremarkable. Besides, a peculiar distortion of the cycle takes place every period just before its "sticking" part. It occurs due to the locally higher velocity of the spring than that of the bow. The potential energy stored by the spring under bowing action at lower velocity part of the period is transformed into kinetic energy, which at relatively low spring mass results in higher spring velocity. At the same time the string force passes locally over the maximum friction force and therefore the string can overtake the bow. This situation, although easily observable in many recorded string waveforms, remains without an appropriate comment in the literature. At any rate, it suggests that the "stick and slip" interpretation of the bowing mechanism, widespread in textbook on musical acoustics, is only valid for a particular range of variables.

The third example put in evidence a possible easy analysis of transient build-up and decay in an acoustic oscillator maintained by jet excitation. This analysis, shown in Fig. 13 and 14, is, of course, oversimplified, because it assumes an abrupt start of the jet flow, as well as an abrupt stop of its action, however, it makes evident a distinctly longer build-up — than a decay-duration.

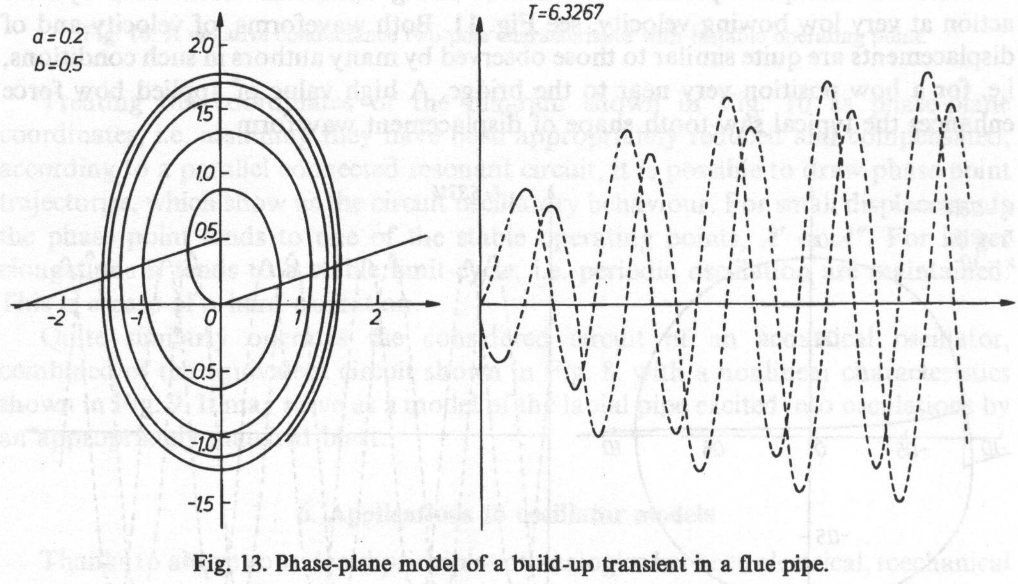

Fig. 13. Phase-plane model of a build-up transient in a flue pipe.

Numerous other examples were studied as models helpful in better understanding and interpretation of the mechanical and acoustical oscillators behaviour, thanks to application of analogies.

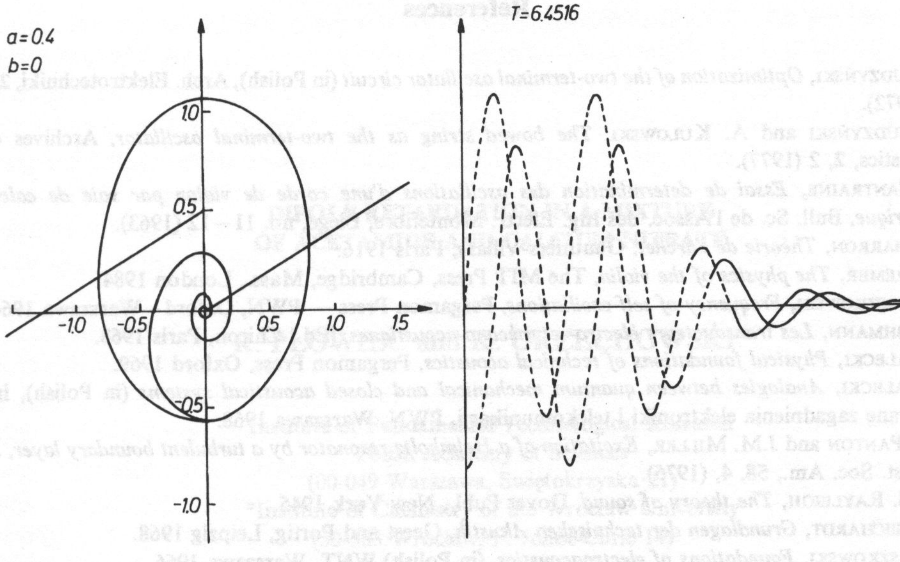

Fig. 14. Phase-plane model of a decay transient in a flue pipe.

7. Conclusions

The presented enlargement of the application of the electroacoustical analogies onto a class of nonlinear two-poles, able to maintain oscillation in resonant circuit, has allowed to analyze easily mechanical and acoustical oscillators. It has been achieved thanks to possibility of comparisons to very well known and easy to investigate properties of electrical oscillators.

The above described approach has afforded new concepts and new interpretations concerning the mechanism of oscillation maintenance in mechanical and acoustical oscillators. Those new ideas may be usefully applied to investigations of self-oscillatory or self-vibratory phenomena occurring in musical instruments. Further applications to studies of wind-borne structure oscillations, of atmospheric oscillations, and of aeroacoustically excited sounds seem to be expected.

A concluding remark concerning the use of analogies may be formulated as follows. The so far employed principles of analogies do not make the most of their possible applications. Further studies should enlarge and precise the above presented concepts. The contributions of Professor I. Malecki to that matter [8, 9] have suggested a fruitful direction of research.

8. Acknowledgement

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