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PROBLEM OF SOURCE MODELLING

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The paper presents a theoretical approach to the problem of modelling of real sound sources of given radiation directivity characteristics. The research concerns only fixed monoharmonic vibrations. The paper presents also the results of experimental concerning the chosen industrial sound sources and the attempts to model these sources by means of a system of substitute sources composed of monopoles. The experimental results can be used for the active sound compensation in the multi-dimensional systems.

1. Introduction

The questions of sound wave radiation emitted by plane sound sources belong to important, but also very difficult problems of technical acoustic. Many of them have not been completely solved theoretically due to the variety of forms and dimensions of the vibrating surfaces and complex distribution of vibration speeds on the surface. The general sound radiation theory makes it in principle possible to define acoustic field distribution around any sound sources. These solutions are however quite formal, because in many practical calculations serious mathematical difficulties occur. They can be overcome only in the case of sources with regular shapes and simple distributions of vibration speeds on the source surface. It results from theoretical considerations that, in case of calculation of the radiation characteristics in each case it is necessary to execute integration on the source surface. When the shape of source surface or the amplitude distribution of vibration speed of the points on the source surface is complex, the integration presents serious difficulties. Use of numerical methods requires, in general, a very dense digitization of the vibrating surface, and forces us to take into consideration mutual influences between the finite elements ΔS. These facts make the problem under consideration even more complicated. Construction of substitute models is another possibility of solving the complex questions of sound source radiation. This method can be efficient only when a real complex source can be replaced by a finite number of sources with simple and known radiation characteristics. Modelling of sound sources is closely connected with active methods of sound abattement.

2. Optimal parameters of substitute sound sources

When the radiation of any sound source is observed from a sufficiently great distance, i.e. from the Fraunhofer zone, the influence of geometrical dimensions of the source can be neglected and the sound source can be treated as quasi-pointwise. In the case of harmonic waves the three-dimensional distribution of acoustic pressure around the real source can be described by the formula [4]

$$p(r,\theta,\varphi)=p_0(r,0,0)\cdot R_0(\theta,\varphi)\ [\mathrm{Pa}] \tag{2.1}$$

Acoustic field generated by a system of substitute quasi-pointwise sources can be presented as a superposition of elementary waves emitted by individual sources.

$$p_z(r,\theta,\varphi)=\sum_{j=1}^{\infty} A_j \frac{e^{-ikr_j}}{r_j} R_j(\theta,\varphi) \quad [\mathrm{Pa}] \tag{2.2}$$

where $A_j \frac{e^{-ikr_j}}{r_j}$ — complex amplitude of the acoustic pressure on the direction of (0, 0); r_j — distance between source j and the observation point.

Directivity indicator of source radiation depends on direction of coordinates. Dependence of radiation directivity indicator on the source localization can be presented in the form [1]:

$$\hat{R}(\theta,\varphi)=R(\theta,\varphi)\exp\left[ik(x_0\cos\varphi\sin\theta+y_0\sin\varphi\sin\theta+z_0\cos\theta)\right], \tag{2.3}$$

where $\hat{R}(\theta,\varphi)$ — radiation directivity indicator of the source translated by relation to origin of coordinates by a distance of vector (x_0, y_0, z_0); $R(\theta,\varphi)$ — radiation directivity indicator of the source situated in the origin coordinates.

By comparing the distribution of acoustic fields generated by a real source and a system of substitute sources it is possible to select the parameters of source system A_n in such a way that the similarity is as strong as possible. For this purpose it is necessary to introduce a similarity criterion. One of the possible criteria is that described by the formula:

$$K=\frac{1}{4\pi A^2}\iint_S |p-p_z|^2 dS, \tag{2.4}$$

where $p=p_0(r,\theta,\varphi)$ — acoustic pressure generated by real source [Pa]; $p_z=p_z(r,\theta,\varphi)$ — total acoustic pressure of a sound source system [Pa]; A — real source moment [Pa·m]; S — surface of sphere with radius r [m^2].

The functional K that was assumed can be compared by analogy with the mean square functional. Integration operation on the spherical surface can be approximated by summation. As a result, the quality functional would be proportional to the sum of squares of acoustic pressure differences. Mean square functional is often used for probability determination owing to well-developed mathematical tools. The assumed criterion has a global nature. It means that the compliance in one chosen direction is not sufficient for the identity of acoustic fields of real source and the system of substitute sources. The simultaneous agreement in all directions is also required. The criterion assumed has also a physical interpretation. It can be interpreted as a relative acoustic power of a system of sound sources composed of a real source and substitute sources vibrating in the opposite phases. By analogy with active this is a relative acoustic power of a system with active noise compensation. Moreover, the assumed quality functional is a dimensionless quantity. In the case of absence of substitute source, the quality functional takes the value of 1, while for an ideal substitute source the functional value is equal 0.

For better interpretation, the concept of a functional level can be introduced in the form

$$L_K = -10\log(K). \tag{2.5}$$

Functional level will assume the values from 0 in the case of absence of a substitute source, up to $+\infty$ in the case of ideal agreement between the real source and the substitute source. Similarity criterion level would correspond to the value of the decrease of acoustic power level for the system with active compensation used in active methods.

Using the functional assumed in that way it is possible to determine the optimal parameters of a system of substitute sound sources. Assuming the notation

$$\begin{aligned} A_{jx} &= \mathrm{Re}\,(A_j), \\ A_{jy} &= \mathrm{Im}\,(A_j), \end{aligned} \tag{2.6}$$

the following equations can be written in the form:

$$\frac{\partial K}{\partial A_{jx}} = 0, \quad \frac{\partial K}{\partial A_{jy}} = 0. \tag{2.7}$$

These equations lead to the linear system of algebraic equations [6]

$$\begin{aligned} \sum_{i=1}^{m} A_{ix}\,U_{ji} - \sum_{i=1}^{m} A_{iy}\,V_{ji} &= U_{j0}, \\ \sum_{i=1}^{m} A_{ix}\,V_{ji} - \sum_{i=1}^{m} A_{iy}\,U_{ji} &= V_{j0}, \end{aligned} \tag{2.8}$$

where

$$\begin{aligned} U_{ji} &= \frac{1}{4\pi r^2}\iint_S (R_i\bar{R}_j + \bar{R}_i R_j)\,dS = U_{ij}, \\ V_{ji} &= \frac{1}{4\pi r^2}\iint_S (R_i\bar{R}_j - \bar{R}_i R_j)\,dS = -V_{ij}. \end{aligned} \tag{2.9}$$

In some cases the integrals (2.9) can be determined in an analytical way. Assuming that the substitute sources are all-directional sources (monopoles), the integrals (2.9) will equal (2.6)

$$U_{ji}=2\,\frac{\sin(kl_{ji})}{kl_{ji}}, \qquad V_{ji}=0, \tag{2.10}$$

where l_{ji} — distance between the all-directional substitute sources [m]; k — wave number [m^{-1}].

Since $V_{ij}=0$, in this case the equation system will be reduced to the form (2.11). This fact considerably simplifies the calculations.

$$\sum_{i=1}^{m} A_{ix}\,U_{ji}=U_{j0}, \qquad \sum_{i=1}^{m} A_{iy}\,V_{ji}=V_{j0}. \tag{2.11}$$

3. Experimental determination of the distribution of acoustic pressure emitted by selected industrial sound sources

The knowledge of real distribution of acoustic pressure around the industrial sound sources is necessary for calculation of parameters of the simplified emission models. For this purpose it is necessary to determine, on the hemispherical surface,

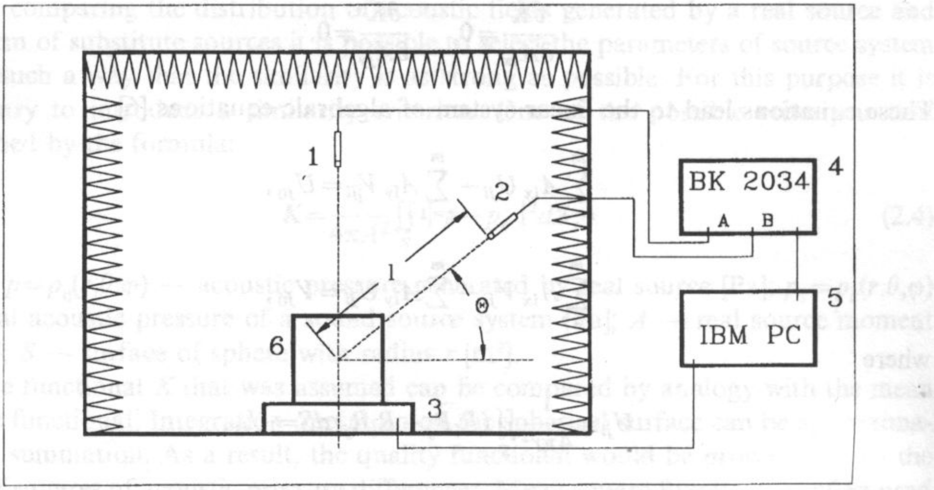

Fig. 1. Scheme of measurement system; *1*, *2* — measurement microphone B&K4134 with preamplifier B&K2619; *3* — turntable B&K3921; *4* — bi-channel signal analyser B&K2034; *6* — tested object.

the distribution of acoustic pressure amplitude, as well as the distribution of angles of phase displacement between the acoustic signals. The measurement system, presented in Fig. 1, was constructed in an anechoic chamber of the Mechanics and Vibroacoustic Institute to determine the acoustic pressure distribution.

The floor in the anechoic chamber was covered by chipboard in order to secure the sound reflecting surface. The sound source tested was situated on the turntable *3*. Acoustic pressure was measured by the microphone *2*. This microphone was placed on a motionless stand at a constant distance $l=1.5$ m from the source. The angle between the line connecting the microphone with the source center and the horizon was assumed in the range from 0 to $\pi/2$ with the step of $\pi/14$. The sound source tested (situated on the turntable) rotated around its own axis. The microphone *1*, situated vertically above the sound source (in a constant direction in relation to the source) was used to compare the vibroacoustic signals. Owing to the simultaneous measurements of the signals from both microphones, it was possible to determine the phase displacements angles between the acoustic pressure occurring in a given direction (determined by the angle θ and the rotation angle of sound source φ) and the reference direction.

The measurements were executed simultaneously in the range from 20 to 20 000 [Hz]. Two-channel analyzer BK2034 was used for the simultaneous analysis of signals arriving from both microphones. Acoustic pressure amplitude was read-out from the proper spectral concentration function of the signal from the microphone *2*. In order to determine the displacement angles between signals, the functions of mutual spectral concentration between signals from microphones *1* and *2* was used. The phase displacement angle was determined from the ratio of imaginary and real parts of that function.

During the determination of acoustic pressure distribution, for each sound source the measurements were carried out in 197 measuring points. In each point 9 measurements were executed, and then the results were averaged. The amplitude of the acoustic pressure was read-out in linear units [Pa], and after averaging the pressure level was determined. By determining the average angle of phase displacement, the average value of the real part and average value of the imaginary part were first determined; then on this basis the angle was determined.

A computer was used for controlling the measurement process. It released the turntable 3 rotation. By the interface IEEE it also controlled the analyser operations. At precisely determined time instances (related with measurement point positions and turntable angular velocity), the computer released the process of registration and analysis of acoustic signals executed by the analyser. After the analysis, the data were read-out and averaged by the computer and then registered on a hard disk. The averaging was executed for 9 turntable rotations. After nine rotations the measurement process was interrupted and the arm of the microphone 2 was displaced (scheme in Fig. 1). Then the entire process was repeated up to the moment when the microphone reached the upper position (angle $= \pi/2$).

The following machines were tested

- industrial vacuum cleaner US4/20 made in Yugoslavia,
- bench grinder SZ-750 P-34/250,
- air compresor 1JS60 made by ASPO Wrocław.

For all the machines the acoustic pressure distribution were determined in frequencies of 200, 300, 400 and 500 [Hz]. The measurement results of the industrial vacuum cleaner obtained for the frequency of 200 Hz are presented in Fig. 2 as an example. The amplitude characteristic of acoustic pressure is presented in a logarithmic scale with dynamics of 50 dB, and the phase characteristic is presented in a linear scale with range $0-2\pi$ [rad].

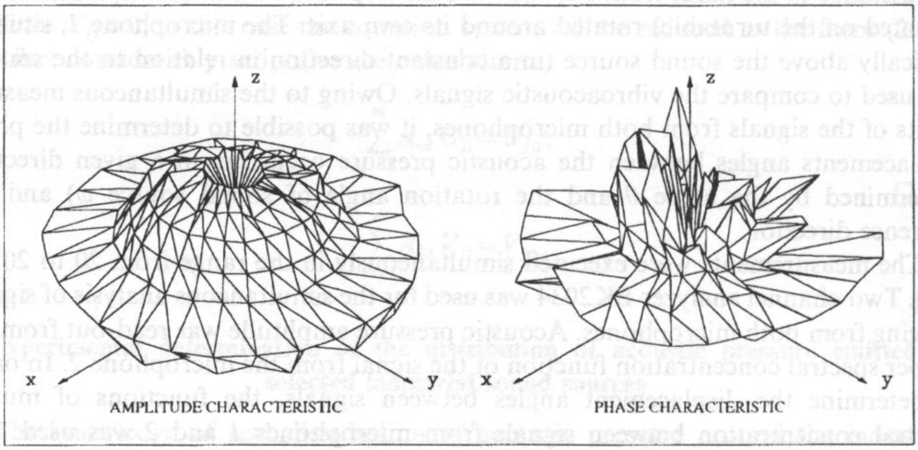

Fig. 2. Radiation directivity characteristic of bench grinder SZ-750 P-34/250. Frequency 200 [Hz].

Owing to the method of result averaging which was applied, small measurement fluctuations of the results were noticed during the experiments. It especially concerned the angle of phase displacement between the signals coming from both microphones. In several frequencies the obtained directivity characteristics of radiation are, however, very irregular. It can be observed, that in the places with significant fluctuation of the acoustic pressure amplitude, great fluctuation of phase displacement angle also occurs. Although the measurements in all frequencies were executed simultaneously, apart from the "wrenched" characteristics, very "smooth" characteristics were also obtained. The experiments were possible owing to application of the system composed of the analyser BK2034 and the computer. The computer was used to control the measurement process as well as the registration and the following result treatment.

4. The optimal arrangement and parameters of substitute sources in emission models of industrial sound sources

On the basis of measurements of acoustic pressure distribution around the chosen industrial sound sources, modelling of acoustic field of real sound sources was executed by means of the acoustic field of all-directional source above the acoustic buffle. The calculations were executed according to the formulas (2.4), (2.10), (2.11). The modelling was done in such a way that the substitute source position was assumed to be within the cuboid with dimensions slightly large than the overall dimensions of the machine tested. For such a position the functional level and the optimal amplitude of substitute source was determined. The simulation for each machine was executed in two planes. The first (horizontal) was at the level of one half of the machine height, the second plane (vertical) ran through the source middle ($y=0$). The calculations for each machine were executed in frequencies 200, 300, 400 and 500 [Hz]. Directivity characteristic of the substitute source was determined by means of Eq. (2.3). An example of such simulation results is presented in Fig. 3.

It results from the research that only a small numbers (usually one) of local maxima of quality functional level and relative optimal amplitude occurs in the examined area. Points at which the local maxima of quality functional levels occurred, overlapped with the points, where maxima of relative optimal amplitude of substitute source occurred. In most of the cases the points of substitute source position for which the quality functional takes the greatest values can be connected with the points of machine structural elements position.

It can also be stated that at the points where quality functional maximum take place, also a maximum of amplitude occurs; however the diagram of optimal amplitude of substitute source are more irregular. It seems that the position of quality functional maximum is related to the position of dominating noise sources in the machine.

For unmistable statement of the connection between the doiminating noise sources in the machine and the maxima of the quality functional, some additional experimental research is necessary. The localization of functional maxima outside the machine can be caused by a noise of aerodynamic origin or the acoustic buffle vibrations.

Modelling of real sound sources by means of several substitute sources can be executed in many different ways. One of the possible methods is presented below. Its algorithm is shown in Fig. 4.

At the beginning it is assumed that the source characteristic is equal to the real source characteristic. It is also assumed that there is one substitute source, positioned in the origin of coordinates. The similarity criterion gradient and the optimal parameters of substitute source are determined. The substitute source is translated towards the maximal gradient direction. This operation is repeated until the local maximum is reached. Then it is verified whether the criterion is greater than required. If not, the source characteristic is changed in accordance with the relationship. The new substitute source situated in the origin of coordinates is also assumed and all the calculations are executed from the beginning, until the similarity criterion reaches the given value. As the result of this procedure, the minimal locations and with optimal parameters. An example of the result of such procedure for air compressor 1JS60 in frequency of 500 [Hz] is presented in Table 1.

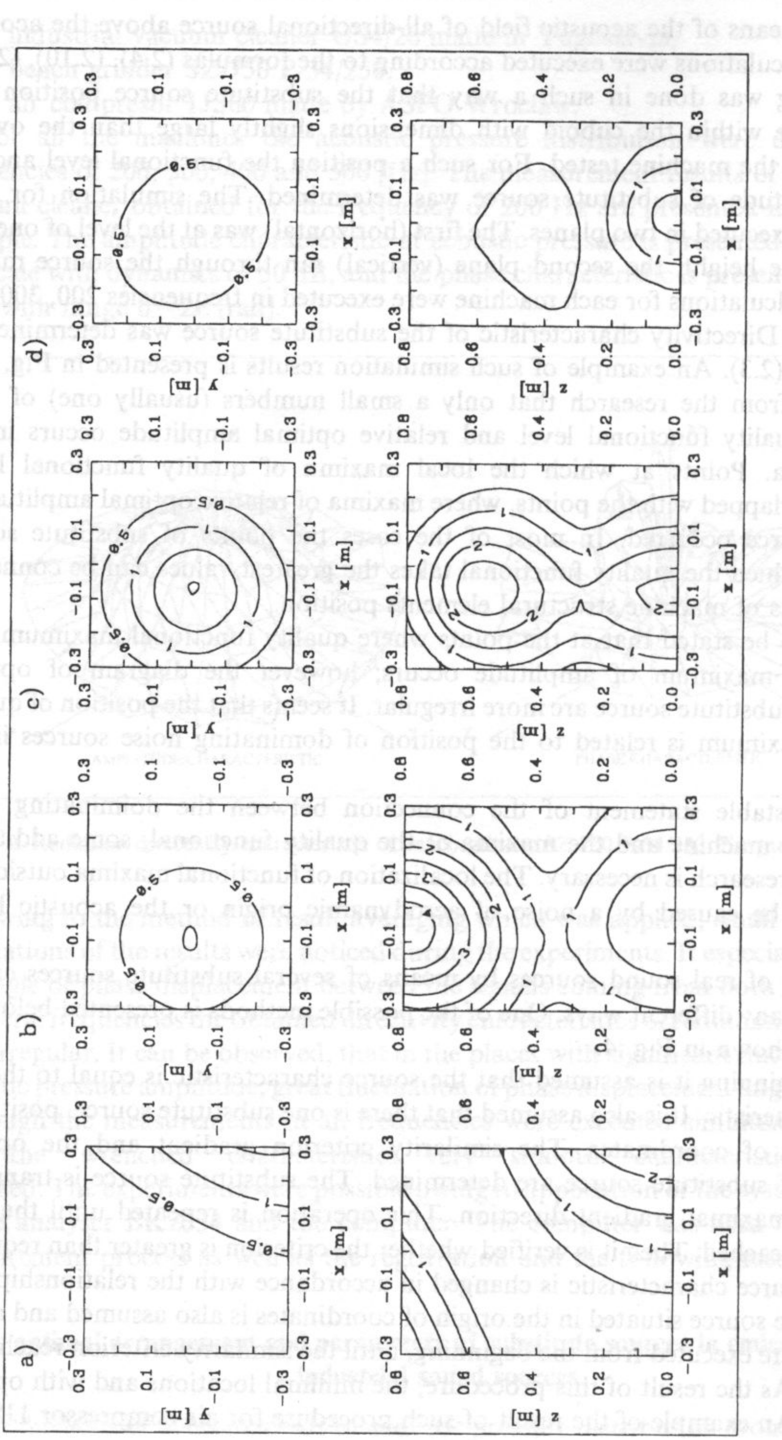

Fig. 3. Quality functional level with the real source replaced by a singular monopole. Industrial vacuum cleaner US4/20 made in Yugoslavia. Frequency: a) 200 [Hz], b) 300 [Hz], c) 400 [Hz], d) 500 [Hz].

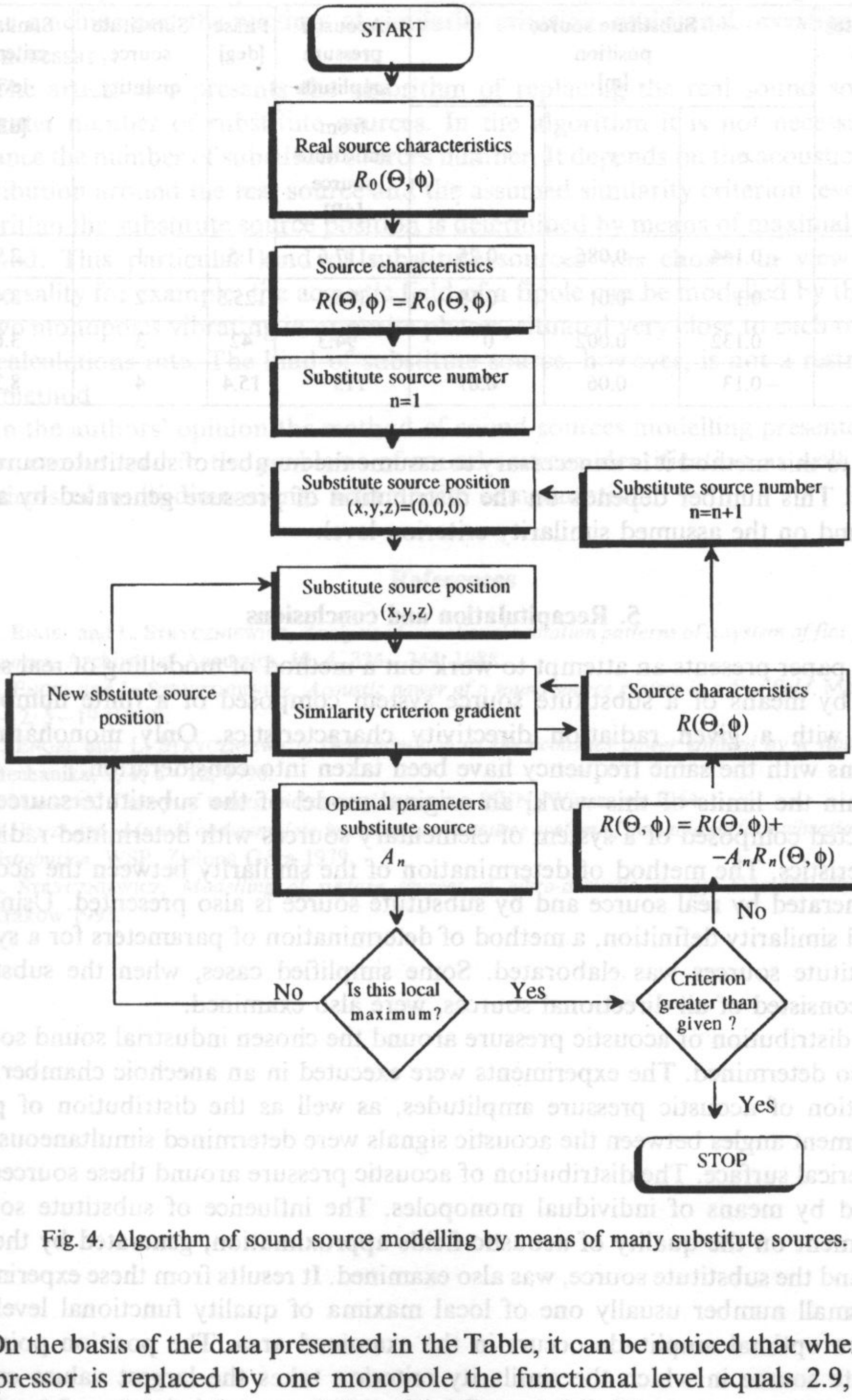

Fig. 4. Algorithm of sound source modelling by means of many substitute sources.

On the basis of the data presented in the Table, it can be noticed that when the air compressor is replaced by one monopole the functional level equals 2.9, by two monopoles — 3.4, by three — 3.6, and by four 8.3 dB.

Table 1

Substitute source number	Substitute source position [m]			Acoustic pressure amplitude from substitute source [dB]	Phase [deg]	Substitute source quantity	Similarity criterion level [dB]
	x	y	z				
1	−0.144	−0.086	0.25	117.6	1.5	1	2.9
2	0.1	−0.01	0.19	105.3	125.5	2	3.45
3	0.132	0.002	0	94.3	42	3	3.63
4	−0.13	−0.06	0.67	113	15.4	4	8.3

Due to this method it is unnecessary to assume the number of substitute sources in advance. This number depends on the distribution of pressure generated by a real source and on the assumed similarity criterion level.

5. Recapitulation and conclusions

This paper presents an attempt to work out a method of modelling of real sound sources by means of a substitute source system composed of a finite number of sources with a given radiation directivity characteristics. Only monoharmonic vibrations with the same frequency have been taken into consideration.

Within the limits of this work, an original model of the substitute source was constructed composed of a system of elementary sources with determined radiation characteristics. The method of determination of the similarity between the acoustic field generated by real source and by substitute source is also presented. Using the assumed similarity definition, a method of determination of parameters for a system of substitute sources was elaborated. Some simplified cases, when the substitute source consisted of all-directional sources, were also examined.

The distribution of acoustic pressure around the chosen industrial sound sources were also determined. The experiments were executed in an anechoic chamber. The distribution of acoustic pressure amplitudes, as well as the distribution of phase displacement angles between the acoustic signals were determined simultaneously on the spherical surface. The distribution of acoustic pressure around these sources was modelled by means of individual monopoles. The influence of substitute sources arrangement on the quality of acoustic fields approximation, generated by the real source and the substitute source, was also examined. It results from these experiments that a small number usually one of local maxima of quality functional level and a relative optimal amplitude occurs in the examined area. The position points of substitute source in which the similarity criterion takes the largest values can be related, in most cases, to localization of the machine structural elements. In order to determine explicitly the quantitative correlation between the dominating noise source in the machine and the maxima of similarity criterion, additional investigations are still necessary.

The article also presents the algorithm of replacing the real sound sources by a greater number of substitute sources. In the algorithm it is not necessary to in advance the number of substitute sources number. It depends on the acoustic pressure distribution around the real source and the assumed similarity criterion level. In this algorithm the substitute source position is determined by means of maximal gradient method. This particular kind of substitute sources was chosen in view of their universality for example: the acoustic field of a fipole can be modelled by the system of two monopoles vibrating in opposite phases, situated very close to each other, and the calculations rate. The kind of substitute source, however, is not a restriction of this method.

In the authors' opinion the method of sound sources modelling presented in this paper can be used in the problems of sound sources identification as well as in the problems of multi-dimensional active noise compensation.

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