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SURFACE ACOUSTIC WAVE DESIGN FUNDAMENTALS

D.C. MALOCHA

Electrical and Computer Engineering Dept.
University of Central Florida
(Orlando, FL 32816-2450, USA)

This paper will provide a fundamental approach to the design of SAW transducers and filters. Although SAW devices are often modeled with complex and large programs, the basic design principles can be analyzed with simple programs and commercially available analysis tools, such as Mathcad or Matlab. The beginning of the paper reviews the basis for the impulse response model. Design equations for fundamental window time functions as applied to apodized transducers will be presented. The development shows an analytical approach to the solution of apodized transducers. The solutions provide the SAW radiating beam profile as a function of frequency as well as the frequency dependent acoustic transducer parameters. The electrical network effects on the overall transducer response are presented which provides a complete first order analysis for fundamental SAW trandsucer design.

1. Introduction

This paper will discuss a fundamental approach to simple but important SAW transducer designs which illustrate the principles in the design process. The principles of finite impulse response design, are not presented in this work. The focus will be on a single transducer design. In it's simplest form, a SAW filter is composed of two transducers which may have different center frequencies, bandwidths, and other filter specifications. The product of two frequency responses produces the total filter response.

The four most popular and widely used SAW models include the transmission line model, the coupling of modes model, the impulse response model, and the superposition model. The superposition model is an extension of the impulse response model and is the principle model used for the majority of SAW bi-directional and multiphase filter synthesis which do not have inband, inter electrode reflections because of its simplicity and ease of understanding. This paper will use the simple impulse response model to illustrate fundamental SAW design principles.

2. The SAW impulse response transducer model

The basic SAW impulse response model can be derived by using energy arguments. To first order, the acoustic impulse response under the transducers is assumed to be sinusoidal having the electrode period as shown in Figure 1. This assumption ignores the actual charge distribution under the electrodes which is proportional to the actual SAW waveform, but yields a good approximation to the fundamental SAW frequency response. First, the single electrode center frequency conductance can be found using energy arguments [1]. The following variables are defined as:

C_s' capacitance per electrode per unit beam width,
W_a acoustic beam width,
E_e stored electric energy,
ΔE_e change in the stored electric energy,
E_a acoustic energy in a single half wavelength,
ΔE_a change in the stored acoustic energy in a single half wavelength,
ΔP_a change in the acoustic power.

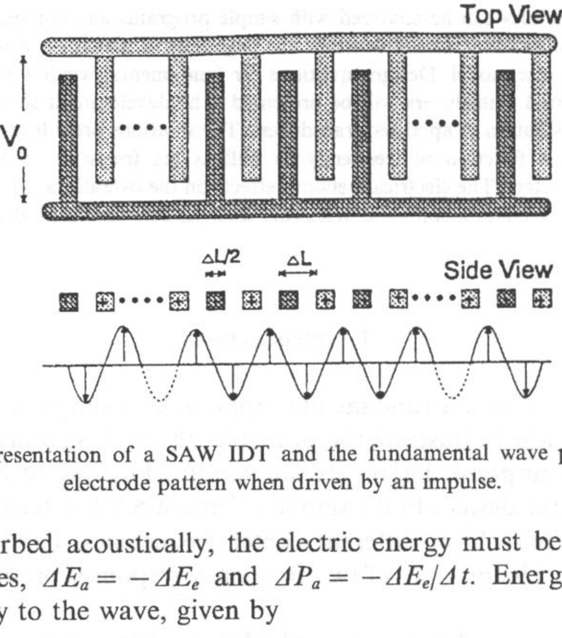


Fig. 1. Schematic representation of a SAW IDT and the fundamental wave perturbation under the electrode pattern when driven by an impulse.

As energy is absorbed acoustically, the electric energy must be decreasing. Then, assuming no losses, $\Delta E_a = -\Delta E_e$ and $\Delta P_a = -\Delta E_e/\Delta t$. Energy will be coupled electro-acoustically to the wave, given by

$$k^2 E_e = \frac{1}{2}\Delta E_a = -\frac{1}{2}\Delta E_e, \qquad (2.1)$$

where k^2 is defined as the acousto-electric coupling coefficient to the wave. The one-half in accounts for the fact that there is both a forward and reverse traveling wave, each containing equal energy. The electric energy transferred to the SAW each half wavelength is

$$E_e = C_s' W_a V_0^2, \tag{2.2}$$

where $C_s' W_a$ is the single electrode transducer capacitance and V_0 is the potential. Assuming no losses and equal static and kinetic energy, the change in acoustic power at center frequency is given by

$$\Delta P_a = -\Delta E_e / \Delta t = 4 f_0 k^2 C_s' W_a V_0^2, \tag{2.3}$$

where $\Delta t = 1/2f_0$ is the period of one-half wavelength and $v_a \Delta t = \Delta L$. From the power relationship, the acoustic conductance per one-half wavelength is given by

$$G_o = \Delta P_a / V_0^2 = 4k^2 f_0 C_s' W_a, \tag{2.4}$$

where C_s' is the capacitance per electrode. Equation (2.4) is the synchronous (or center frequency) acoustic conductance for a single half wavelength (or single electrode) for the transducer.

For a linear causal system, the inverse Fourier transform of the device's frequency response is the device time impulse response. The SAW impulse response for an uniform beam transducer with arbitrary electrode spacing can be written as

$$h(t) = A_0 \cos[\theta(t)) \operatorname{rect}(t/r) \qquad \text{where } \theta(t) = 2\pi \int_0^t f_i(\tau) d\tau \tag{2.5}$$

and where $f_i(t)$ equals the instantaneous frequency at a time, t, and A_0 equals a constant. For a uniform beam transducer with periodic electrode spacing, $f_i(t) = f_0$ and $\cos\theta(t) = \cos\omega_0 t$.

Given the form of the time response, energy arguments are used to determine the device equivalent circuit parameters for a transducer of arbitrary length. Assume a delta function voltage input, $v_0(t) = \delta(t)$, then $V_0(\omega) = 1$. Given $h(t)$, $H(\omega)$ is known and the energy launched as a function of frequency is given by $E(\omega) = 2 \cdot |H(\omega)|^2$ [2]. Then

$$E(\omega) = V_0^2(\omega) \cdot G_a(\omega) = 1 \cdot G_a(\omega), \tag{2.6}$$

or

$$G_a(\omega) = 2 \cdot |H(\omega)|^2. \tag{2.7}$$

There is a direct relationship between the transducer frequency transfer function and the transducer conductance. Consider an IDT with uniform overlap electrodes having N_p interaction pairs. Each gap between alternating polarity electrodes is considered a localized SAW source. The SAW impulse response at the fundamental frequency will be continuous and of duration τ, where $\tau = N \cdot \Delta t$, and $h(t)$ is given by

$$h(t) = A_0 \cdot \cos(\omega_0 t) \cdot \operatorname{rect}(t/r), \tag{2.8}$$

where ω_0 is the carrier frequency, in radians per second.

The corresponding frequency response is given by

$$H(\omega) = \frac{A_0 \tau}{2} \left\{ \frac{\sin(x_1)}{x_1} + \frac{\sin(x_2)}{x_2} \right\}, \tag{2.9}$$

where $x_1 = (\omega - \omega_0) \cdot \tau/2$ and $x_2 = (\omega + \omega_0) \cdot \tau/2$.

The ideal SAW continuous response in both time and frequency is now known. This can be related to the sampled response by a few substitutions of variables, let

$$\Delta t = \frac{1}{2 \cdot f_0} \qquad t_n = n \cdot \Delta t \qquad N \cdot \Delta t = \tau \qquad N_p \cdot \Delta t = \tau/2, \tag{2.10}$$

where N is the total number of electrodes (or half wavelengths) and N_p is the total number of electrode pairs. Assuming a frequency bandlimited response, the negative frequency component centered around $-f_0$ can be ignored. Then the frequency response, using (2.9) is given by

$$H(\omega) = A_0 \left\{ \frac{N}{4f_0} \right\} \cdot \frac{\sin(x_n)}{x_n}, \tag{2.11}$$

where $x_n = \dfrac{(\omega - \omega_0)}{\omega_0} \pi N_p$. The conductance, given using (2.7) and (2.9), is

$$G_a(f) = \frac{A_0^2}{8} \left\{ \frac{N}{f_0} \right\}^2 \cdot \frac{\sin^2(x_n)}{x_n^2} = \frac{A_0^2}{2} \left\{ \frac{N_p}{f_0} \right\}^2 \cdot \frac{\sin^2(x_n)}{x_n^2}. \tag{2.12}$$

A_0 can be found solving (2.12) using (2.4) at center frequency and with $N = 1$. Then

$$A = (32k^2 f_0^3 C_s' W_a)^{1/2} \qquad \text{for } N = 1, \tag{2.13}$$

and

$$A_N [(32k^2 f_0^3 C_s' W_a N^2)^{1/2}, \qquad \text{for all } N, \tag{2.14}$$

where A_N represents the peak wave amplitude under the transducer for N electrodes. It is typical to define the wave amplitude per wavelength (or electrode pair), given as

$$A_{Np} = (4k^2 f_0^3 C_s W_a N_p^2)^{1/2}, \tag{2.15}$$

where the electrode pair capacitance is $C_s = 2C_s'$. The peak wave amplitude is proportional to the number of electrodes (or impulse length) at the synchronous center frequency. The energy within the confines of the transducer area is given as $E = A_{N_p}^2$. The energy exiting the transducer from either end is $\frac{1}{2}E$, therefore, the peak wave amplitude for either the forward or reverse traveling wave is $A_{N_p}\sqrt{2}$.

The center frequency conductance is given from (2.12) as

$$G_a(f_0) = G_0 = 8k^2 f_0 C_s W_a N_p^2 \tag{2.16}$$

or the frequency dependent transducer conductance is

$$G_a(f_0) = G_0 \cdot \frac{\sin^2(x_n)}{x_n^2}. \tag{2.17}$$

The transducer electrode capacitance is given as

$$C_e = C_s W_a N_p \, . \tag{2.18}$$

Finally, the last term of the SAW transducer's equivalent circuit is the frequency dependent susceptance. Given any system where the frequency dependent real part is known, there is an associated imaginary part which must exist for the system to be real and causal. This is given by the Hilbert transform susceptance, defined as B_a, where [3]

$$B_a(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{G_a(u)}{(u-\omega)}\,du = G_a(\omega) * 1/\pi\omega \, , \tag{2.19}$$

where "*" indicates convolution.

The simple impulse model treats each electrode as an ideal impulse, however, the electrodes have a finite width which distorts the ideal impulse response. The actual SAW potential has been shown to be closely related to the electrostatic charge induced on the transducer by the input voltage and the details will not be presented here [4].

3. Apodized SAW transducers

Apodization is the most widely used method for weighting a SAW transducer, as depicted in Figure 2. The desired time sampled impulse response is implemented by assigning the overlap of opposite polarity electrodes at a given position to a normalized sample weight at a given time. Typical computer analysis divides the acoustic beam into a number of parallel tracks where the impulse response can be represented, to any required accuracy, as the summation of uniform samples located at the proper positions in time in a given track. Mathematically this is given by

$$h(t) = \sum_{i=1}^{I} h_i(t) \, ,$$

and

$$H(\omega) = \sum_{i=1}^{I} H_i(\omega) = \sum_{i=1}^{I} \left\{ \int_{-\tau/2}^{\tau/2} h_i(t) e^{-j\omega t} dt \right\} . \tag{3.1, 3.2}$$

Although this approach is general, exact analytical solutions are obtainable for some important types of simple functions which provide insight into the spatial distribution of the energy in an apodized transducer acoustic beam.

A mapping can be obtained from time to space for some simple but useful time functions. For illustration, let's assume a cosine envelope time function given by

$$h(t) = A \cdot \cos(\pi t/\tau)\cos(\omega_0 t)\,\mathrm{rect}(t/\tau)) \, .$$

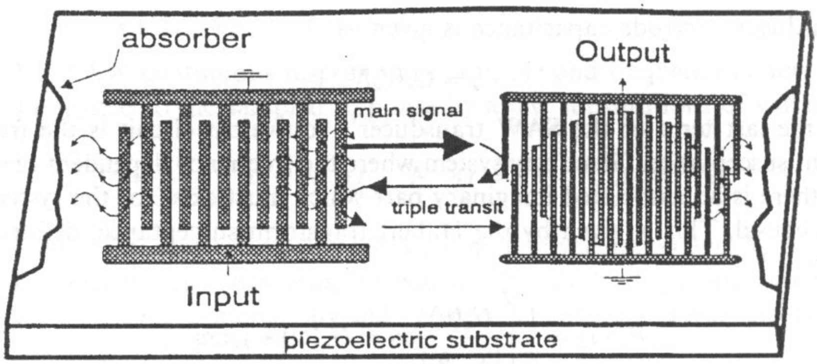


Fig. 2. Schematic diagram of a typical SAW filter composed of one unweighted interdigital transducer (IDT) and an apodized transducer.

Defining the y-direction as the propagation direction and the x-direction as transverse to the propagation direction, then a mapping can be made between the functional description and the time impulse response lenght as a function of x which describes $h(t, x)$. Looking at the apodization pattern at each point in x shows that the impulse response length is centered at time $t = 0$ and the time response is

$$h(t, x) = A \cdot \cos(\omega_0 t) \cdot \mathrm{rect}(t/\tau_1(x)),$$

where $\tau_1(x)$ is the impulse response lenght at each point in x.

The impulse response length as a function of x is given as

$$\tau_i(x) = \frac{2 \cdot \tau}{\pi} \cdot \mathrm{arc}\cos\left(\frac{2 \cdot |x|}{W_a}\right).$$

The form of the solution has the familiar Fourier transform

$$H(x, f) = \frac{A \cdot \tau_1(x)}{2} \cdot \left\{ \frac{\sin[y_1(x, f)]}{y_1(x, f)} + \frac{\sin y_2(x, f)]}{y_2(x, f)} \right\},$$

where

$$y_1(x, f) = \frac{f - f_0}{f_0} 2\pi f_0 \frac{\tau_1(x)}{2} \quad \text{and} \quad y_2(x, f) = \frac{f + f_0}{f_0} 2\pi f_0 \frac{\tau_1(x)}{2}.$$

This yields the spatially dependent frequency response which allows the amplitude profile emerging from the transducer to be plotted. The amplitude profile spatially varies and is dependent on the frequency of interest as well as the fractional bandwidth. The overall frequency response is given by

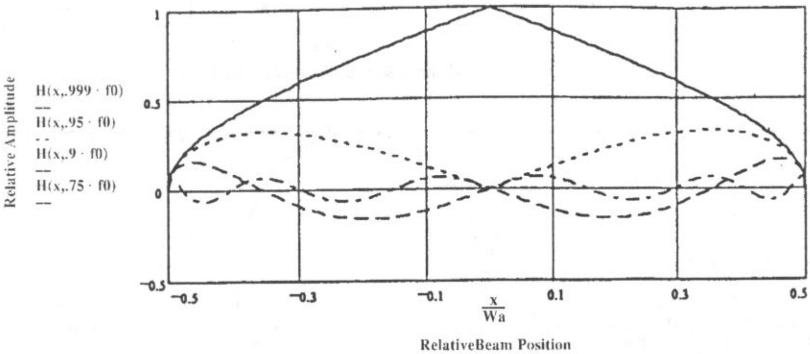
$$H(f) = A \cdot \int_{-Wa/2}^{Wa/2} H(x, f)\, dx.$$

Figure 3 shows the transducer frequency response and the amplitude beam profile for several frequencies. As another example, an inverse cosine time function is given by

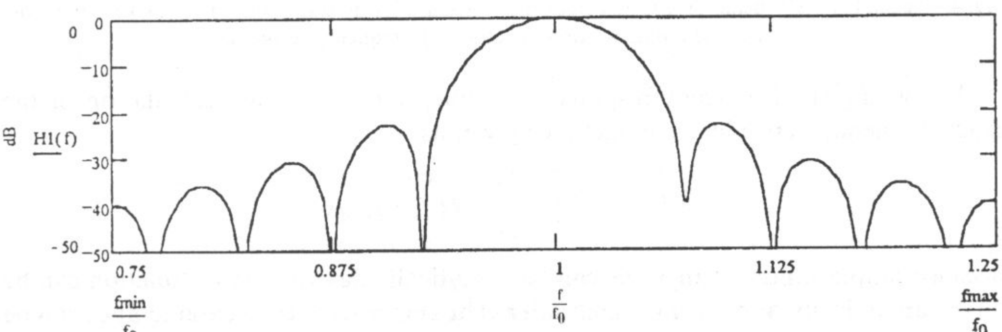
$$h(t) = \frac{2 \cdot A}{\pi} \cdot \arccos(2|t|/\tau)\cos(\omega_0 t)\mathrm{rect}(t/\tau).$$

and the spatially dependent time function is given as

$$h(t, x) = \arccos(\omega_0 t)\,\mathrm{rect}\,(t/\tau_2(x)] \quad \text{where} \quad \tau_2(x) = \frac{\tau}{2} \cdot \cos(\pi x / Wa).$$



a) Cosine Window Function



b) Frequency Response

Fig. 3. Calculated SAW beam profile and frequency response for cosine envelope time function. a) Cosine window function; b) Frequency response.

The spatially dependent amplitude response as a function of frequency is shown in Figure 4. Notice that the amplitude profile is much smoother at the beam center than for the cosine envelope time function. The inverse cosine time function is often used in resonator structures because of its smoother amplitude beam profile and maximum energy confinement to the beam center. This analysis is also useful if examining transverse mode coupling in a waveguide.

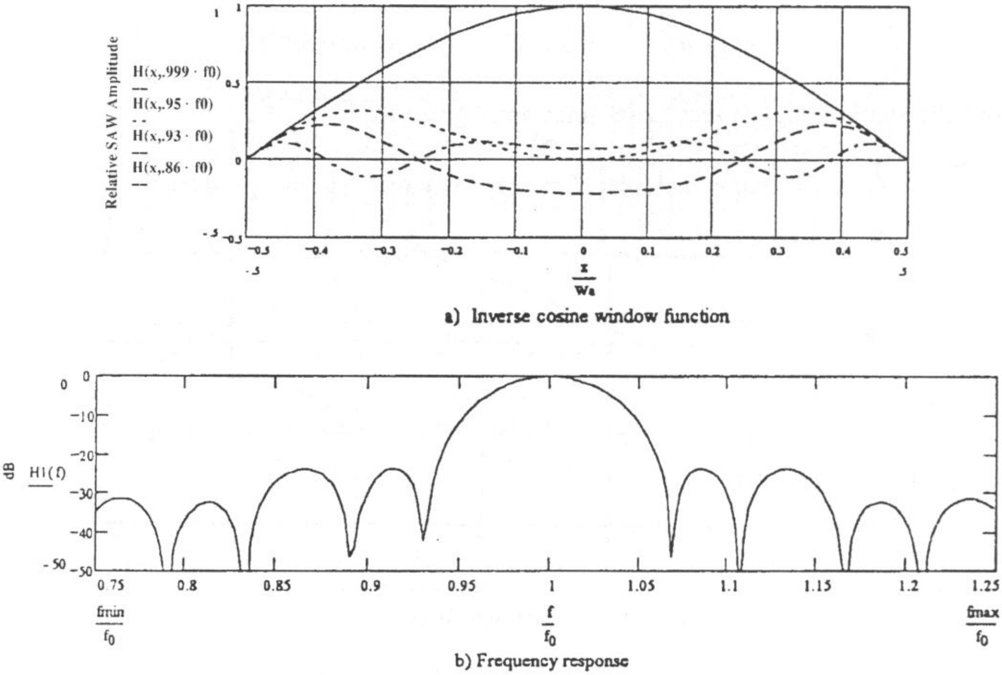


Fig. 4. Calculated SAW beam profile and frequency response for inverse cosine envelope time function. a) Inverse cosine window function; b) Frequency response.

Knowning the frequency response as a function of x allows calculation of the exact frequency dependent conductance given as

$$G_a(f) = 2\int_{-Wa/2}^{Wa2/2} |H(f, x)|^2 dx.$$

Because simple window functions can be analytically described, this function can be sloved analytically or by using a computer. The center frequency conductance can be written using the previous results and sloving the integral as

$$G_a(f_0) = 8k^2 f_0 C_s W_a N_{\text{eff}}^2.$$

N_{eff} is a function of the impulse response lenght and the form of the time function. For the transverse cosine time function $N_{\text{eff}} = \dfrac{N_p}{\sqrt{2}}$ and for the cosine time envelope $N_{\text{eff}} = .68 * N_p$. The plot of the conductance and frequency response are given in Figure 5. Note that there is energy radiated even at the nulls of the frequency response due to the apodization effect.

There is also a secondary effect of apodization when attempting to extract energy. Not all of the power of a non-uniform SAW beam can be extracted by an a uniform transducer, and reciprocally, not all of the energy of a uniform SAW beam can be extracted by an apodized transducer. The transducer efficiency is calculated at center frequency for an analytic function as

$$E=\frac{\left|\int_{-Wa/2}^{Wa/2} H(f_0,\ x)dx\right|^2}{Wa\int_{-Wa/2}^{Wa/2} |H(f_0,\ x)|^2 dx}$$

and the apodization loss is defined as: apodization loss $= 10\cdot\log(E)$. For the transverse cosine function the apodization loss is .921 dB and for the cosine time function the apodization loss is .571 dB.

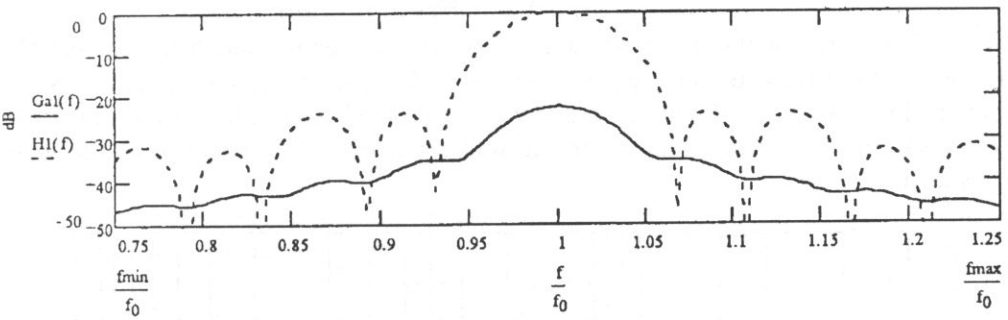


Fig. 5. Plots of the calculated frequency response and acoustic conductance for an apodized inverse cosine time function. The center frequency-impulse responde lenght product equals 20.

4. Electrical network effects

Given the acoustic conductance and Hilbert transform susceptance, and the transducer static capacitance, the SAW transducer equivalent circuit is known, shown in Figure 6, and the electrical network effects can be determined. Define the transducer Q and the load Q, respectively, as

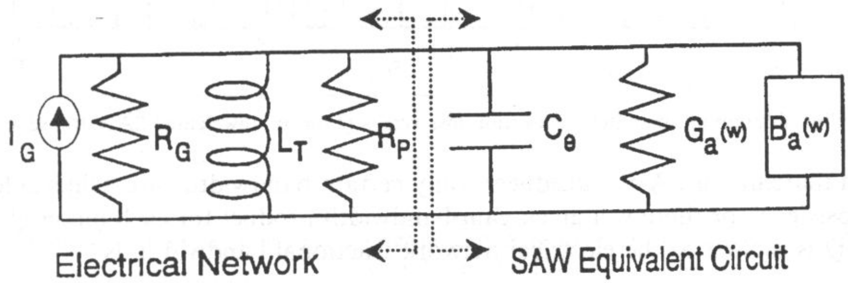


Fig. 6. SAW equivalent circuit model which includes the generator, parasitic resistance, and a tuning inductor.

$$Q_r = \frac{\omega_0 C_3 Wa}{Ga(f_0)} \quad \text{and} \quad Q_g = \frac{\omega_0 C_3 Wa}{G_g},$$

where C_g is the generator conductance. Then the center frequency transducer gain can be written as

$$G_T = \frac{4Q_g/Q_r}{(1+Q_g/Q_r)^2 + Q_g^2}.$$

The minimum unmatched insertion loss can be found for a given transducer Q with a certain generator Q by taking the derivative of G_T and setting it to zero which yields.

$$Q_G = \sqrt{\left[\frac{Q_r^2}{1+Q_r^2}\right]}.$$

Figure 7 is a plot of the minimum unmatched transducer mismatch loss versus Q_r. To obtain the lowest unmatched insertion loss, for low Q transducers it is best to match the real part of the transducer to the load resistance while for high Q transducers it is best to match the transducer's reactive impedance to the load resistance.

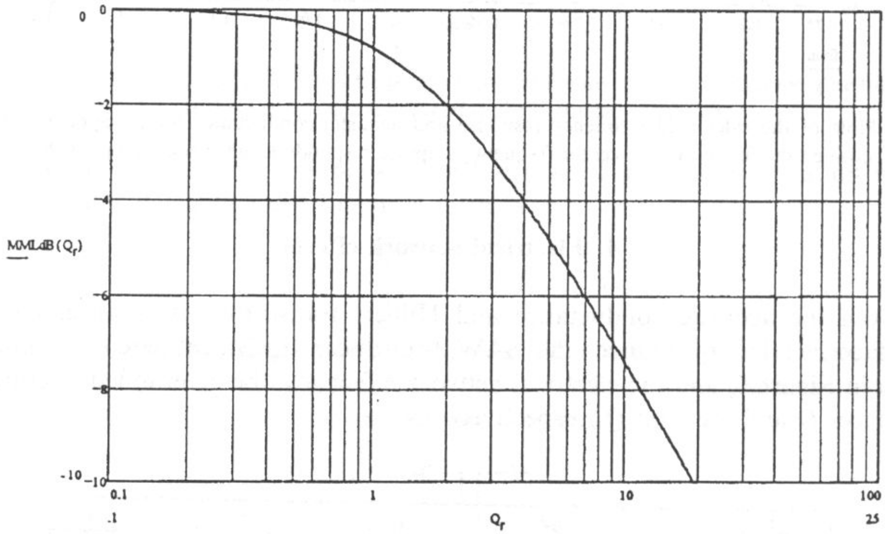


Fig. 7. Plot of minimum unmatched insertion loss versus Q for an unweighted SAW transducer.

When matching a SAW transducer, only certain bandwidths are achievable for a given loss level, or there is a given gain-bandwidth product for each material. The network Q is related to the electrical network fractional bandwidth as

$$Q_r = \frac{\omega_0 C_3 Wa}{2 \cdot Ga(f_0)} = f_0/\Delta f_e.$$

For minimum electrical distortion it is usually assumed that the electrical fractional bandwidth is twice the acoustical bandwidth, or $\Delta f_e = 2\Delta f$. Then back substituting from the previous equations and assuming a uniform weighted IDT, yields

$$\frac{\omega_0 C_3 Wa}{2\cdot Ga(f_0)} = \frac{\pi/8k^2}{N_p} = \frac{f_0}{2\Delta f} \quad \text{or} \quad \{\Delta f/f_0\}_{\max} \leqslant \sqrt{4k^2/\pi}\,.$$

If the fractional bandwidth is larger than given above, then the transducer insertion loss increases at a rate of 6 dB per octave.

The effects of the electrical network can be demonstrated by assuming a simple parallel matching inductor, and no parasitic resistance. It will be assumed that the transducer can be exactly matched to the real load impedance. The transfer function, ignoring the Hilbert transform susceptance can be written as

$$H_e(\omega) = \frac{G_g/\omega_0 C_e}{\dfrac{\beta G_a(f_0)}{\omega_0 C_e} + \dfrac{G_a(f)}{G_a(f_0)} + \left[\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega}\right]}, \quad \text{where } \beta = G_g/G_a(f_0)\,.$$

Figure 8 shows a series of plots of the effects of the electrical network transfer function as a function of Q. At center frequency, half the voltage is on the SAW conductance, which corresponds to the -6 dB level. Off center frequency, the voltage increases which causes a loss in sidelobe rejection.

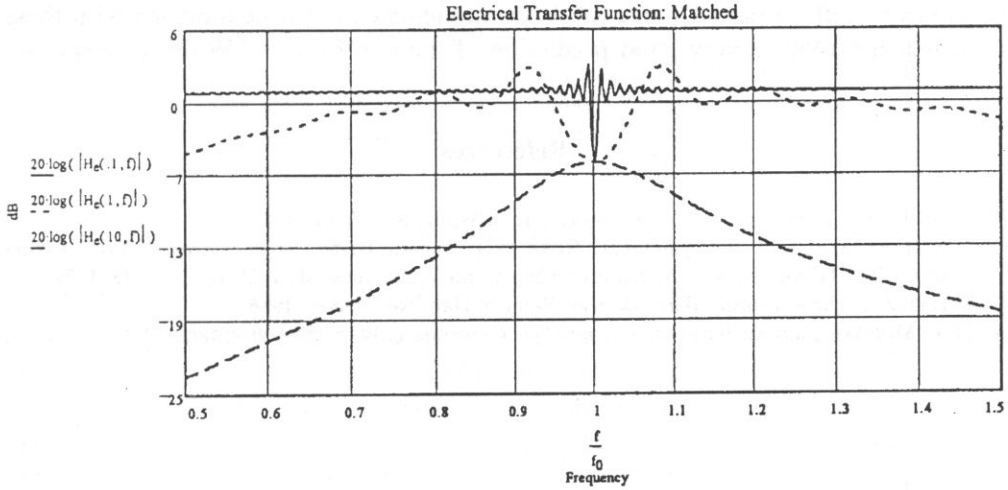


Fig. 8. Electrical network transfer function versus frequency for SAW transducer Q_s of 0.1, 1, 10, respectively, for $\beta = 1$.

The effect on the overall filter response is shown in Fig. 9, which shows a degradation in the sidelobe levels and a slight distortion of the bandwidth.

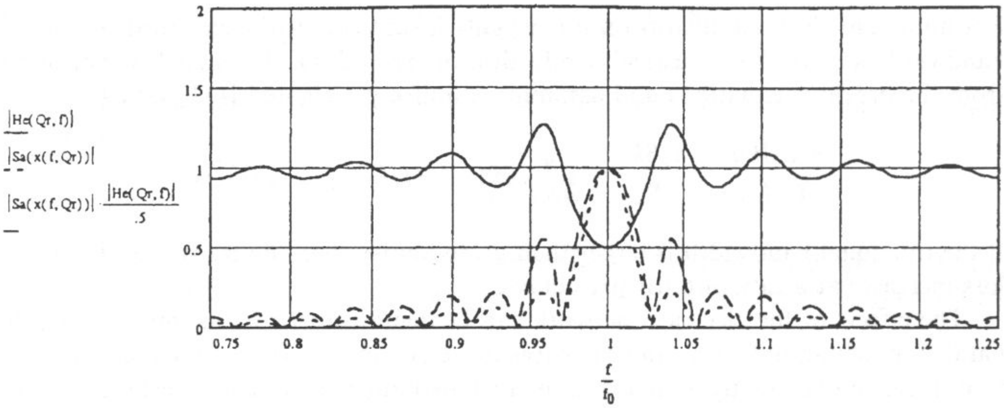


Fig. 9. The effects of the electrical network for a SAW transducer with a Q of 0.5. The solid line is the voltage transfer function, the dotted line is the ideal sin x / x response and the dashed line is the product response.

5. Conclusion

This paper has presented a simple approach to the first order design of SAW filters, including the apodization effect. All of the figures and plots were obtained using Mathcad, which is a very affordable tool for accomplishing first order analysis of elementary SAW transducers. More complicated structures, such as multiphase transducers, reflectors and other SAW components can also be modeled with these tools which provide a very good prediction of the first order SAW device response.

References

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